Continuous Transitions in Mereotopology

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Abstract

Continuity from a qualitative perspective is different from both the philosophical and mathematical view of continuity. We explore different intuitive notions of spatio-temporal continuity. We present a general formal framework for continuity and continuous transitions in mereotopology for spatio-temporal histories and thus sketch the correctness of the conceptual neighbourhood for the qualitative spatial representation language RCC-8.

1 Introduction

We want to formalize the *intuitive* notion of spatio-temporal continuity for a qualitative theory of motion. We consider temporally extended regions in space. This ontological view is not entirely new (e.g. see [Russell, 1914; Carnap, 1958; Clarke, 1981]). More recently, [Hayes, 1985; Vieu, 1991] have considered all objects to be occurrent and regarded as spatio-temporal (henceforth: s-t) histories. To the best of our knowledge [Muller, 1998a] is the first attempt at a full mereotopological theory of space-time.

However it is worth noting that continuity from a qualitative perspective is different from both philosophical and mathematical view of continuity. Even though we commit to an ontology where objects are occurrent, we do not attempt a formal characterization of the identity criteria, which is difficult [Wiggins, 1980] and also beyond the scope of this paper. The problem of continuity of continuants still lacks a convincing treatment. There are a number of possibilities in the literature to cope with this (see [Thomson, 1983]). Some involve considering four dimensional space-time (e.g. [Heller, 1990]) while others focus on a revised theory of parts [Simons, 1987].

Muller presents an intuitive notion of s-t continuity and one that is perhaps nearest to a qualitative understanding of motion. Apart from Muller, the main work which addresses what continuity implies for a common-sense theory of motion is [Galton, 2000]. However, it falls short of an explicit generic characterization of s-t continuity in a pointless mereotopology.

Intuitive spatio-temporal continuity (as previously proposed by [Muller, 1998a]) is temporal continuity without spatial leaps. However, such a notion of continuity allows *temporal pinching* i.e., a history is allowed to disappear and re-appear instantaneously and *weird* transitions are possible (see fig. 10). To avoid temporal pinching, we introduce a notion of *firm-connectedness*. We investigate the different notions of s-t continuity and transitions possible under distinct notions and provide a hierarchy of conceptual neighbourhood diagrams.

Further, Muller's interpretation of history-based theorems of transition has been shown not to be fully adequate [Davis, 2000]. Davis analyses the conditions under which Muller's theory can be said to be adequate and presents an alternative more comprehensive framework for characterisation of transitions. However, it is not expressed as an object level first order theory, and it sacrifices the spirit of mereotopology as it defines time instants and s-t points[1].

Davis claims that proving the correctness of rules that state the non-existence of transitions or worse, the existence of transitions from plausible mereotopological axioms would seem to be daunting if not hopeless. Taking up this challenge, we characterize transitions in pure mereotopology over s-t regions. In order to identify an instantaneous relation occurring during a transition between histories we present an exhaustive categorization of relationships between adjacent parts of histories. Under the strongest notion of s-t continuity, we axiomatize continuous transitions for maximal firmly-connected s-t histories. We sketch how to use the formulation to recover the RCC-8 conceptual neighbourhood [Cohn *et al.*, 1997]. Our approach is closer to Muller's than to Davis' in that we present

[1] [Pratt and Schoop, 1998] argue that points can always be defined and their non introduction is thus illusory. However the explicit introduction of points is still counter to the original motivation and spirit of mereotopology [Whitehead, 1929; Gerla, 1995]. Moreover it is possible there may be computational reasons to eschew their explicit introduction. We recognize that whether points are allowed or not in a mereotopology is perhaps controversial, but we believe that at the very least it is interesting to explore the possibility of not introducing them at all.

a "naive-physical" theory, rather than one closely based on mathematical topology.

2 Mereo-Topological Framework

We will use three connection relations: $\mathsf{C}_{\mathsf{st}}, \mathsf{C}_{\mathsf{sp}}$ and C_{t} for spatio-temporal, spatial and temporal connection respectively. The interpretation of these relations is as shown[2] in Fig 1.

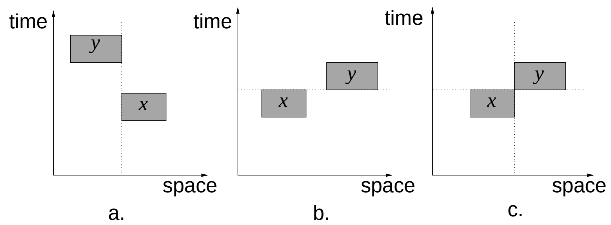

Figure 1: **a**. *Spatial* **b**. *Temporal* and **c**. *Spatio-Temporal* connection between two spatio-temporal entities x and y.

The binary relation of spatio-temporal connection $\mathsf{C}_{\mathsf{st}}xy$: x is spatio-temporally connected to y is true just in case the closures of x and y at least share a s-t point[3] – fig. 1c. Spatial connection for space-time entities is their connection in pure space. As shown in Fig. 1a connection under spatial projection is interpreted along the temporal axis i.e., spatial connection on projection to an infinitesimally thin *temporal slice* at right angles to the temporal axis. Spatial connection is written as $\mathsf{C}_{\mathsf{sp}}xy$: x is spatially connected to y – x and y are s-t regions whose closures have a spatial point in common, though not necessarily simultaneously. Finally temporal connection is $\mathsf{C}_{\mathsf{t}}xy$: x and y are s-t regions whose closures have a temporal point in common, though not necessarily at the same place. Fig. 1b shows temporal connection between spatio-temporal regions x and y.

The axiomatisation of these connection relations are identical and follows [Cohn *et al.*, 1997]. Note that in this theory the closure and its interior cannot be distinguished. We have the following axioms:

A1. $\mathsf{C}_\alpha xx$
A2. $\mathsf{C}_\alpha xy \rightarrow \mathsf{C}_\alpha yx$
A3. $[\forall z(\mathsf{C}_\alpha zx \leftrightarrow \mathsf{C}_\alpha zy) \leftrightarrow (x =_\alpha y)]$

where $\alpha \in \{\mathsf{st}, \mathsf{sp}, \mathsf{t}\}$[4].

[2]Space is shown as 1D in these illustrations and the others figures in the paper, but this is simply for ease of drawing. The defined concepts are applicable to 2D and other higher dimensional space.

[3]Note that although we use points in the informal semantics here, this does not mean we are introducing them at the object level. Moreover there are other interpretations which do not involve points at all, e.g. a distance metric [Randell *et al.*, 1992], or Boolean Connection Algebras [Stell, 2000].

[4]For clarity at times we omit the subscript α from predicates. In such cases we mean $\alpha = \mathsf{st}$ (i.e., unless stated otherwise we mean a spatio-temporal subscript).

For the sake of clarity, throughout the paper universal quantifiers scoping over whole formulas are omitted. Lower case symbols in *italics* stand for variables whereas predicates are stated a priori.

2.1 Mereo-Topological Relations

From $\mathsf{C}_\alpha xy$ we can define the mereological relation of parthood, $\mathsf{P}_\alpha xy$: x is a part of y.

D1. $\mathsf{P}_\alpha xy \equiv_{def} \forall z(\mathsf{C}_\alpha zx \rightarrow \mathsf{C}_\alpha zy)$

The parthood relation is used to define *proper-part* (PP_α), *overlap* (O_α) and *disjoint* (DR_α). Further, $\mathsf{DC}_\alpha, \mathsf{EC}_\alpha, \mathsf{PO}_\alpha, \mathsf{EQ}_\alpha, \mathsf{TPP}_\alpha$ and NTPP_α i.e., *disconnected, externally connected, partial overlap, equal, tangential proper part* and *non-tangential proper part* respectively can be defined. These relations, along with the inverses for the last two viz. TPPi_α and NTPPi_α, constitute the eight JEPD (jointly exhaustive and pairwise disjoint) relations of RCC-8 (see [Cohn *et al.*, 1997] for definitions). Fig. 2a show the JEPD set of RCC-8 relations in space-time, whereas Fig. 2b is the equivalent relations under C_{sp}.

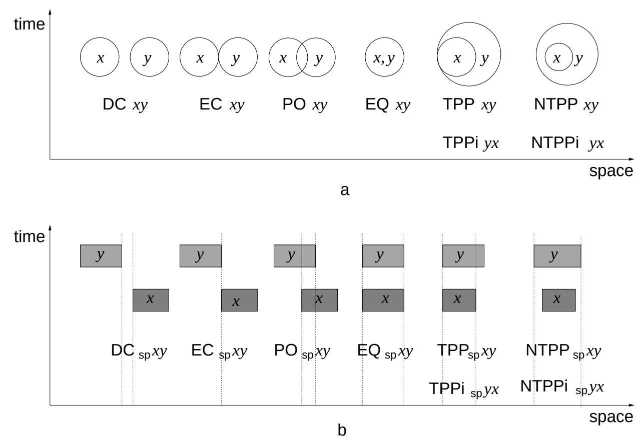

Figure 2: **a**. JEPD set of RCC-8 relations in space-time formed from C_{st} and **b**. RCC-8 relations under C_{sp} connection between two entities x and y.

We introduce the following existential axioms. Axiom A4 ensures every region has a nontangential part. In A5 the individual z is noted $x \cup y$ or $x + y$ and represents the sum, whereas in A6 it is noted $x - y$ and represents the difference. In A7 the individual z represents the intersection and is noted as $x \cap y$.

A4. $\forall y \exists x \mathsf{NTPP} xy$
A5. $\exists z \forall u(\mathsf{C}uz \leftrightarrow (\mathsf{C}ux \vee \mathsf{C}uy))$
A6. $\mathsf{PO}xy \rightarrow \exists z \forall w((\mathsf{P}wx \wedge \mathsf{DR}wy) \leftrightarrow \mathsf{P}wz)$
A7. $\mathsf{O}xy \rightarrow \exists z \forall u(\mathsf{C}uz \leftrightarrow \exists v(\mathsf{P}vx \wedge \mathsf{P}vy \wedge \mathsf{C}vu))$

To identify instantaneous relations between histories (such as in Figure 12) in a pointless mereo-topology requires the categorization of relation between certain parts of histories. This requires a notion of connection different from the straightforward s-t connection. We will introduce the notion of *firm connection*. A firm-connection in n-D space is defined as a connection wherein an n-D worm can pass through the connection without becoming visible to the exterior. Thus, for two regions to be firmly-connected a direct *conduit* exists between the two [Cohn and Varzi, 1999]. Figure 3 illustrates firm connection and non-firm connection.

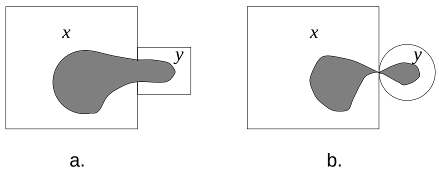

Figure 3: **a**. *Firm* and **b**. *Non-Firm* connection between two entities x and y.

In order to define firm-connection, we define one-piece or spatio-temporal connectedness. A spatio-temporal region is spatio-temporally one-piece, $\mathsf{CON}_{\mathsf{st}}x$ just in case all parts of x are C_{st} connected. Similarly to represent that a certain temporal extent is one-piece we define temporal connectedness : spatio-temporal region x is temporally one-piece just in case all parts of x are temporally connected. We can also define spatial connectedness: spatio-temporal region x is spatially one-piece just in case all parts of x are C_{sp} connected. We have the following definition.

D2. $\mathsf{CON}_\alpha x \ \equiv_{def} \ \forall y, z(x = (y+z) \rightarrow \mathsf{C}_\alpha)$

D4 states that a connection between two entities x and y is a firm-connection just in case some one-piece part of x ($\mathsf{CON}_{\mathsf{st}}x$) and some one-piece part of y ($\mathsf{CON}_{\mathsf{st}}y$) is interior connected ($\mathsf{INCON}(x+y)$).

D3. $\mathsf{INCON}x \ \equiv_{def} \ \forall y, z, v[((x = y+z) \wedge \mathsf{NTPP}vy) \rightarrow \exists w(\mathsf{P}vw \wedge \mathsf{NTPP}wx \wedge \mathsf{O}wz \wedge \mathsf{CON}w)]$

D4. $\mathsf{FCON}xy \ \equiv_{def} \ \exists u, v[\mathsf{P}ux \wedge \mathsf{P}vy \wedge \mathsf{CON}u \wedge \mathsf{CON}v \wedge \mathsf{INCON}(u+v)]$

In defining transitions between RCC relations, it will be helpful to treat RCC relations as constant symbols rather than as predicates; thus we define a predicate $\mathsf{rcc}_\alpha(\psi, x, y)$: meaning Ψ_α holds between s-t regions x and y (where ψ is the lowercase translation of the RCC relation Ψ).

D5. $\mathsf{rcc}_\alpha(\psi, x, y) \equiv \Psi_\alpha(x, y)$

2.2 Temporal Relations

At times for clarity, we will write the temporal relations as infix operators [Muller, 1998b]. Therefore temporal connection $\mathsf{C}_\mathsf{t}xy$: x is temporally connected to y is also written as $x \asymp y$. We will also write $\mathsf{P}_\mathsf{t}xy, \mathsf{PO}_\mathsf{t}xy$ and $\mathsf{EQ}_\mathsf{t}xy$ as $x \subseteq_\mathsf{t} y, x\ \sigma_\mathsf{t}\ y$ and $x \equiv_\mathsf{t} y$ respectively.

In order to introduce a spatio-temporal interpretation we must capture a notion of temporal order between the entities of the theory. Following [Muller, 1998a] we write $x < y$ for temporal order meaning the closure of x strictly precedes the closure of y in time. Axiom A8 establishes that temporal connection and temporal order are incompatible. Also temporal order is anti-symmetric (A9). Axiom A10 establishes the composition of temporal connection and temporal order.

A8. $x \asymp y \rightarrow \neg x < y$

A9. $x < y \rightarrow \neg y < x$

A10. $(x < y \wedge y \asymp z \wedge z < w) \rightarrow x < w$

Allen [Allen, 1984] and even before him Nicod [Nicod, 1924] pointed out that if time is totally ordered then there are 13 JEPD (jointly exhaustive and pairwise disjoint) relations in which one *one-piece* interval can stand to another which can be defined in terms of *meets*. We give the definition for *meets* (D6) which is a specialization of EC_t and define relations that we will be using in subsequent formulations. D7 is the definition for one interval ending with another and D8 for one interval starting with another. D9 states interval x to be the interval between two distinct intervals y and z. Fig. 4 shows the different temporal relations.

D6. $x \bowtie_\mathsf{t} y \ \equiv_{def} \ \mathsf{EC}_\mathsf{t} \wedge \neg\exists v_1, v_2(v_1 \subseteq_\mathsf{t} x \wedge v_2 \subseteq_\mathsf{t} y \wedge v_2 < v_1)$

D7. $x \sqsupset|_\mathsf{t}\ y \ \equiv_{def} \ \forall u(x \bowtie_\mathsf{t} u \leftrightarrow y \bowtie_\mathsf{t} u)$

D8. $x\ |\sqsubset_\mathsf{t}\ y \ \equiv_{def} \ \forall u(u \bowtie_\mathsf{t} x \leftrightarrow u \bowtie_\mathsf{t} y)$

D9. $x \ \|_\mathsf{t}\ (y; z) \ \equiv_{def} \ (y \bowtie_\mathsf{t} x \wedge x \bowtie_\mathsf{t} z)$

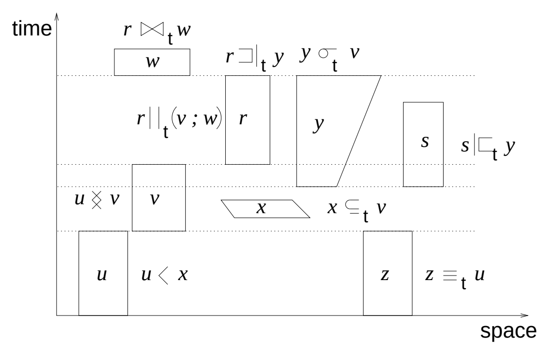

Figure 4: Temporal relations for space-time regions.

2.3 Spatio-Temporal Relations

A s-t connection implies a spatial as well as a temporal connection. Though note that the converse is not necessarily true. Fig 5 shows spatio-temporal regions x and y are spatially[5] and temporally connected but not spatio-temporally. Therefore we have the following axiom:

A11. $\mathsf{C}xy \rightarrow (x \asymp y \wedge \mathsf{C_{sp}}xy)$

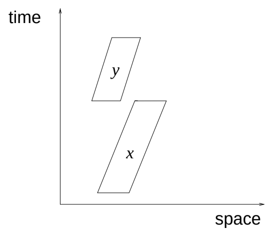

Figure 5: x and y $\mathsf{C_{sp}}$ and $\mathsf{C_t}$ but do not $\mathsf{C_{st}}$

We also introduce the notion of a 'temporal slice', i.e., the maximal component part corresponding to a certain time extent [Muller, 1998a].

D10. $\mathsf{TS}xy \equiv_{def} \mathsf{P}xy \wedge \forall z((\mathsf{P}zy \wedge z \subseteq_{\mathsf{t}} x) \rightarrow \mathsf{P}zx)$

Henceforth, the notation $\frac{y}{w}$ denotes the part of y corresponding to the lifetime of w when it exists (i.e., when $w \subseteq_{\mathsf{t}} y$)[6]. Fig. 6 shows the "temporal slice" and also the when x is not a "temporal slice" of y for a part of x is missing.

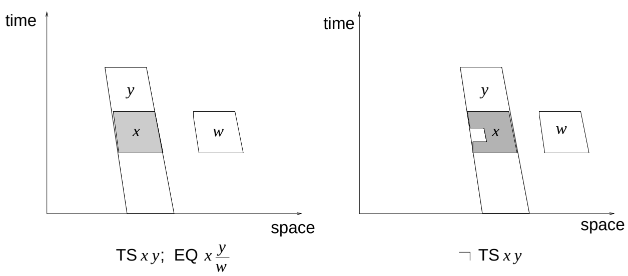

Figure 6: Temporal slice $\mathsf{TS}xy$ and when x is not a temporal slice of y

We introduce relationships to refer to the initial and final parts of a history. D11 states that a part of a history y can be termed its initial part just in case it starts with y and ends before it. Conversely, x is the final part of a history y (D12) just in case x starts after y and ends with it.

D11. $\mathsf{IP}xy \equiv_{def} \mathsf{P}xy \wedge x \mid\!\sqsubset_{\mathsf{t}} y \wedge \exists z(z \sqsupset\!|_{\mathsf{t}}\, y \wedge x \bowtie_{\mathsf{t}} z \wedge x \cup z = y)$

D12. $\mathsf{FP}xy \equiv_{def} \mathsf{P}xy \wedge x \sqsupset\!|_{\mathsf{t}}\, y \wedge \exists z(z \mid\!\sqsubset_{\mathsf{t}} y \wedge z \bowtie_{\mathsf{t}} x \wedge x \cup z = y)$

[5]Recall that spatial connection is interpreted as connection of spatial projections onto an infinitesimally thin temporal slice at right angles to the temporal axis.

[6]The notation $\frac{y}{w}$ is purely syntactic sugar: any atom $\alpha(..\frac{y}{w}..)$ could equivalently be replaced by $\forall x(\mathsf{TS}xy \wedge x \equiv_{\mathsf{t}} w) \rightarrow \alpha(..x..)$

Finally, models must not be spatio-temporal alone, so spatio-temporal connection $\mathsf{C_{st}}$ needs to be different from temporal as well as spatial connection.

A12. $\exists x \exists y \quad \mathsf{C_t}xy \wedge \neg \mathsf{C_{st}}xy$

A13. $\exists x \exists y \quad \mathsf{C_{sp}}xy \wedge \neg \mathsf{C_{st}}xy$

3 Space-Time Continuity

The notion of continuity should implicitly capture the intuitive notion of motion and this is the notion that has been addressed in the existing literature on qualitative continuity mentioned above. However, various weaker notions are possible, and we will explore these below (these bear a strong relationship to the various notions of connection in [Cohn and Varzi, 1999]). First we need to define the notion of a x being a component of a region y, i.e. if it is a maximal one piece part:

D13. $\mathsf{Comp}xy \equiv_{def} \mathsf{CON}x \wedge \mathsf{P}xy \wedge \forall w[[\mathsf{CON}w \wedge \mathsf{P}wy] \rightarrow w = x]$

We now define various notions of what it means for a history to be continuous. First of all consider the case of the history having but a single component. This is essentially the case considered by Muller [7] who defines the notion of a history being continuous if it is temporally self-connected and it doesn't make any spatial leaps:

D14. $\mathsf{CONT}w \equiv_{def} \mathsf{CON}_t w \wedge \forall x \forall u((\mathsf{TS}xw \wedge x \asymp u \wedge \mathsf{P}uw) \rightarrow \mathsf{C}xu)$

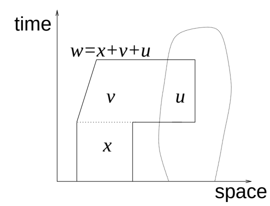

Figure 7: The region w is discontinuous under Muller's definition of continuity because it makes a 'sideways spatial leap'.

See fig. 7 for an illustration of discontinuity under this definition. However, this definition of continuity permits "temporal pinching" of histories – that is histories may disappear and reappear again instantaneously at the same spatial location. We can define a stronger notion of continuity for histories and disallow temporal pinching which we term *firm continuity*. A *non pinched* continuous space-time history is firmly continuous.

Figure 8a shows a firm-connected history w, while Figure 8b is for a history with "temporal pinching". D15 is the definition of a non-pinched history w and D16 defines firm-continuity.

[7]Muller uses a slightly different definition of $\mathsf{CON}_t w$ because his language distinguishes the interiors and closures of regions.

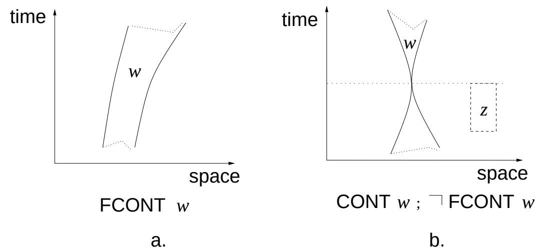

Figure 8: **a**. *Firmly-continuous* history and **b**. A *non-firm* history with instantaneous *temporal pinching* at the end of z.

D15. $\mathsf{NP}w \ \equiv_{def}\ \neg\exists x\exists y[\mathsf{P}xw \wedge \mathsf{P}yw \wedge x \bowtie_{\mathsf{t}} y \wedge \neg\mathsf{FCON}xy]$

D16. $\mathsf{FCONT}w \ \equiv_{def}\ \mathsf{CONT}w \wedge \mathsf{NP}w$

If a history contains multiple components, then we can consider how these relate to each other over time. The strongest notion, which corresponds directly to the case of a single component history is if all components are equi-temporal:

D17. $\mathsf{StrCONT}w \ \equiv_{def}\ \forall x[\mathsf{Comp}xw \rightarrow x \equiv_{\mathsf{t}} w]$

If not all components endure for the time of the whole history, then we can isolate several cases. Firstly, further components may come into existence (a kind of *multiplication*), but once they start, they carry on until the end, as do the original component(s) (e.g. the urban landscape where spatially disjoint new cities may be formed from "green fields" and then never revert away being urban):

D18. $\mathsf{MulCONT}w \ \equiv_{def}\ \exists x[\mathsf{Comp}xw \wedge x \equiv_{\mathsf{t}} w] \wedge \forall y[\mathsf{Comp}yw \rightarrow \mathsf{FP}yw]$

There is a natural dual to this, where all components start simultaneously but some may finish early (a kind of *collapse*, e.g. the gold deposits on the planet earth, which become fewer in number as they are mined and seams become exhausted):

D19. $\mathsf{ColCONT}w \ \equiv_{def}\ \exists x[\mathsf{Comp}xw \wedge x \equiv_{\mathsf{t}} w] \wedge \forall y[\mathsf{Comp}yw \rightarrow \mathsf{IP}yw]$

We can still regard a history as having a weak notion of continuity providing there is at least one component which lasts the entire time:

D20. $\mathsf{WCONT}w \ \equiv_{def}\ \exists x[\mathsf{Comp}xw \wedge x \equiv_{\mathsf{t}} w]$

Still weaker, we may allow for the possibility that no component endures for the entire history, i.e. there may be spatial jumps providing there are no temporal gaps (this might correspond to the history of a particular species of plant in which colonies die out, but others meanwhile are formed):

D21. $\mathsf{TCONT}w \ \equiv_{def}\ \neg\exists x[\mathsf{Comp}xw \wedge x \equiv_{\mathsf{t}} w] \wedge \forall z[z \subseteq_{\mathsf{t}} w \rightarrow \exists y \mathsf{P}yw/z]$

Dually, we may imagine that a history is spatially continuous, in the sense that its components spatially overlap, but there may be temporal gaps (e.g. a lake or river which dries up periodically):

D22. $\mathsf{SpCONT}w \ \equiv_{def}\ \neg\exists x[\mathsf{Comp}xw \wedge x \equiv_{\mathsf{t}} w] \wedge \forall xy[[\mathsf{Comp}xw \wedge \mathsf{Comp}yw] \rightarrow \mathsf{C_{sp}}xy]$

A (weaker) variant would be that for any component of w, the next component (or all of them if more than one component starts simultaneously) is $\mathsf{C_{sp}}$. All these notions of continuity are illustrated in fig. 9.

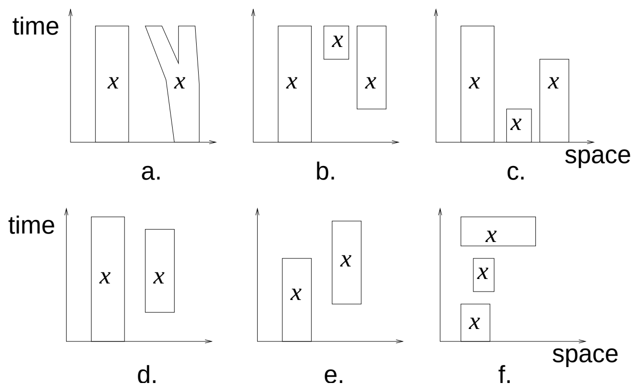

Figure 9: Six kinds of continuous history: **a**. StrCont **b**. MulCont **c**. ColCont **d**. WCont **e**. TCont **f**. SpCont

Each of these notions of continuity can be further refined to exclude temporal pinchings and also spatial leaps within a component. We do not have the space to investigate all of these notions in detail here. Our principal focus will be strong continuity with no temporal pinchings and no spatial leaps:

D23. $\mathsf{StrFCONT}w \ \equiv_{def}\ \mathsf{StrCONT}w \wedge \mathsf{NP}w \wedge \forall x\forall u((\mathsf{TS}xw \wedge x \between u \wedge \mathsf{P}uw) \rightarrow \mathsf{C}xu)$

For convenience of reference this notion of continuity for histories will be labelled $\mathfrak{C}\mathbb{S}$-0. Allowing temporal pinching weakens $\mathfrak{C}\mathbb{S}$-0 to $\mathfrak{C}\mathbb{S}$-1 or $\mathfrak{C}\mathbb{S}$-2 depending on whether temporal pinching of one or both the histories involved in transition between relations of the spatial representation language RCC-8 is allowed.

3.1 Continuous Transitions

With $\mathfrak{C}\mathbb{S}$-0 the intuitive transitions between histories hold. Under s-t interpretations for RCC relations and with "temporal pinching", we can have a number of weird transitions (e.g. as shown in the fig. 10). Fig. 10a shows the transition from EC to TPP. This is possible for the "temporal pinching" of history y. In fig. 10b both the histories x and y undergo "temporal pinching" and a transition from EC to EQ results.

The transition networks for $\mathfrak{C}\mathbb{S}$-0, $\mathfrak{C}\mathbb{S}$-1 and $\mathfrak{C}\mathbb{S}$-2 are shown in fig. 11. It can be seen how weakening the notion

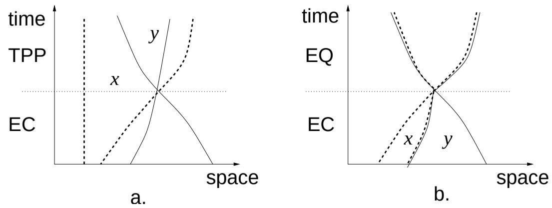

Figure 10: Transition from **a**. EC to TPP and **b**. EC to EQ.

of continuity adds direct transition links to the conceptual neighbourhoods.

Note that the diagram for $\mathfrak{CS}$-2 differs slightly from the conceptual neighbourhood given in fig. 10 of [Davis, 2000], e.g. his figure has a direct link from DC to TPP. This depends on the interpretation of the spatial relationship holding when regions pinch to a spatial point. Davis considers the normalised (regularised) spatial cross section and isolated points will thus disappear, leading to the introduction of yet further links. We could also take this approach in which case his fig. 10 and our diagram for $\mathfrak{CS}$-2 should be identical.

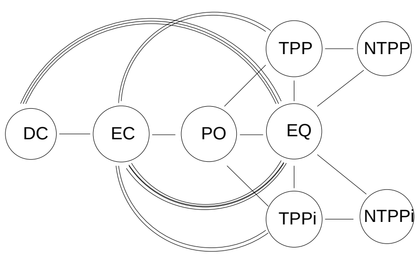

Figure 11: Transition graph for $\mathfrak{CS}$-0, $\mathfrak{CS}$-1 and $\mathfrak{CS}$-2. Additional links for $\mathfrak{CS}$-1 are double arcs and for $\mathfrak{CS}$-2 are triple arcs.

[Galton, 2000] identifies transitions as durative or instantaneous depending on whether the times involved are intervals or instants and whether the initial and final states are separated by an interval or an instant and defines eight different transition operators. In order to describe the different transitions in our mereotopology, we define two durative transition operators (see fig. 13a and 13b) and one which identifies the case where a relationship only holds instantaneously (fig. 12).

Recognizing and classifying durative transitions such as in Fig. 13(a,b) from EC to DC or vice versa is relatively straightforward in mereotopology. The problem occurs when a RCC relation only holds instantaneously, e.g. a transition from PO to NTPP with TPP holding instantaneously in between (Fig. 12). In a true, pointless, mereotopology, there is no direct way to represent the fact that x and y are TPP instantaneously in the above example. However, we will show in the subsection immediately below how this relationship can be identified without direct appeal to points, by categorizing the mereotopological relations between temporally adjacent parts of histories.

A Model for Instantaneous Relations

In this subsection we analyse from first principles which relations can hold instantaneously and under what conditions. The underlying hypothesis for our analysis is that it is sufficient to consider the Boolean combinations of two regions and their FCON relationship over the instantaneous transition.

We will thus determine the existence of an instantaneous topological relation between two histories x and y occurring when two intervals z_1 and z_2 meet, based upon the comparison of $(x \cup y)$, $(x \cap y)$, $(x - y)$ and $(y - x)$, restricted to the intervals z_1 and z_2 respectively. These can be combined such that they form 16 fundamental descriptions:

$$\begin{bmatrix} [\frac{x\cup y}{z_1}|\frac{x\cup y}{z_2}]_{11} & [\frac{x\cup y}{z_1}|\frac{x\cap y}{z_2}]_{12} & [\frac{x\cup y}{z_1}|\frac{x-y}{z_2}]_{13} & [\frac{x\cup y}{z_1}|\frac{y-x}{z_2}]_{14} \\ [\frac{x\cap y}{z_1}|\frac{x\cup y}{z_2}]_{21} & [\frac{x\cap y}{z_1}|\frac{x\cap y}{z_2}]_{22} & [\frac{x\cap y}{z_1}|\frac{x-y}{z_2}]_{23} & [\frac{x\cap y}{z_1}|\frac{y-x}{z_2}]_{24} \\ [\frac{x-y}{z_1}|\frac{x\cup y}{z_2}]_{31} & [\frac{x-y}{z_1}|\frac{x\cap y}{z_2}]_{32} & [\frac{x-y}{z_1}|\frac{x-y}{z_2}]_{33} & [\frac{x-y}{z_1}|\frac{y-x}{z_2}]_{34} \\ [\frac{y-x}{z_1}|\frac{x\cup y}{z_2}]_{41} & [\frac{y-x}{z_1}|\frac{x\cap y}{z_2}]_{42} & [\frac{y-x}{z_1}|\frac{x-y}{z_2}]_{43} & [\frac{y-x}{z_1}|\frac{y-x}{z_2}]_{44} \end{bmatrix}$$

Each element $[\beta|\gamma]_{ij}$ of the matrix represents a condition $\pi_{ij}(\beta, \gamma)$. We will call this matrix $\mathsf{IM}^{\pi}(r, x, y, z_1, z_2)$, where π is a 4x4 matrix which gives the 16 predicates which form the conditions. The entire matrix, IM is to be regarded as a conjunction of its elements:

D24. $\mathsf{IM}^{\pi}(r, x, y, z_1, z_2) \equiv_{def} \bigwedge_{i=1}^{4}[\bigwedge_{j=1}^{4} \mathsf{IM}_{ij}(x, y)]$

The notion of 'firm-connection' between the 16 individual pairs was identified as a simple test [8] that enables the identification of whether an instantaneous relationship occurs. Thus each π_{ij} is either FCON or $\neg$FCON.

Constraints for Non-Existing Relations

Based on the (FCON) or ($\neg$FCON) outcome of each pair, 2^{16} possibilities exist for the instantaneous relation matrix; however only a small number of them are possible. The aim of this section is to make *explicit* the possibilities that are not feasible, thus arriving at the ones that characterize the class of instantaneous relations between two given histories[9]. The way we will achieve this is to consider what restrictions can be placed on the various matrix elements (in terms of their FCONnectivity) in order to be sure that the transition is indeed instantaneous. Thus in each of the conditions we specify which impossible values for π and thus for $\mathsf{IM}^{\pi}(r, x, y, z_1, z_2)$ can be excluded. For notational convenience, in the matrices below, an entry at position i,j of ϕ means FCON whilst $\neg\phi$ means $\neg$FCON.

Condition 1 *The union of the two histories before and after*

[8] In case of parts of a pair not existing for one of the intervals the connection is assumed to be $\neg$FCON without any loss in generality of the analysis.

[9] Those relations that are irrelevant for a given condition and, thus can take any of the two values will be marked by a *wild card* (-).

an instantaneous transition is always FCON.

$$\pi \neq \begin{bmatrix} \neg\phi & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

If the intersection of the two histories $(x \cap y)$ doesn't disappear instantaneously, the intersection and the union is always FCON. Further, the difference (i.e., $(x - y)$ and $(y-x)$) between the histories is related to the amount of intersection. Thus, if it goes out of existence instantaneously the difference between the histories would also disappear instantaneously. The following two conditions are based on this property.

Condition 2 *The union-intersection pair is equivalent to the intersection-intersection pair for two histories before and after an instantaneous transition.*

$$\pi \neq \begin{bmatrix} \phi & \mathsf{A2} & - & - \\ \mathsf{B1} & \mathsf{B2} & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix} where\ \mathsf{A2} \neq \mathsf{B1} \neq \mathsf{B2} \neq \mathsf{A2}.$$

Condition 3 *If the intersection-intersection pair is* ¬FCON*, all others except the union-union pair are* ¬FCON*.*

$$\pi \neq \begin{bmatrix} \phi & \neg\phi & - & - \\ \neg\phi & \neg\phi & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

Conditions 4 to 9 are for firm-connection of the union-intersection i.e, the intersection not disappearing instantaneously. These are based on the property of maximal connected histories and the underlying assumption that pairs with parts that do not exist in either one of the intervals cannot have a firm-connection.

Condition 4 *Both the difference pairs cannot be* FCON *simultaneously.*

$$\pi \neq \begin{bmatrix} \phi & \phi & - & - \\ \phi & \phi & - & - \\ - & - & \phi & - \\ - & - & - & \phi \end{bmatrix}$$

The following two conditions state that for the difference pair being FCON, the union-difference pairs before and after needs to be simultaneously FCON (recall that the union-union is always FCON). This condition is required to be stated separately for $(x - y)$ and $(y - x)$ for the difference is asymmetric.

Condition 5 *If the difference pair $(x - y)$ is* FCON*, the union-difference pairs for $(x - y)$ cannot be* ¬FCON*.*

$$\pi \neq \begin{bmatrix} \phi & \phi & \neg\phi & - \\ \phi & \phi & - & - \\ \phi & - & \phi & - \\ - & - & - & - \end{bmatrix} \wedge \pi \neq \begin{bmatrix} \phi & \phi & \phi & - \\ \phi & \phi & - & - \\ \neg\phi & - & \phi & - \\ - & - & - & - \end{bmatrix}$$

Condition 6 *If the difference pair $(y - x)$ is* FCON*, the union-difference pairs for $(y - x)$ cannot be* ¬FCON*.*

$$\pi \neq \begin{bmatrix} \phi & \phi & - & \neg\phi \\ \phi & \phi & - & - \\ - & - & - & - \\ \phi & - & - & \phi \end{bmatrix} \wedge \pi \neq \begin{bmatrix} \phi & \phi & - & \phi \\ \phi & \phi & - & - \\ - & - & - & - \\ \neg\phi & - & - & \phi \end{bmatrix}$$

The next two conditions follow the same justification as stated for 5 and 6, but for the difference before and after the instantaneous transition being ¬FCON. Under such circumstances the union-difference pairs cannot be FCON.

Condition 7 *If the difference pair $(x - y)$ is* ¬FCON*, the union-difference pairs for $(x - y)$ cannot both be* FCON*.*

$$\pi \neq \begin{bmatrix} \phi & \phi & \phi & - \\ \phi & \phi & - & - \\ \neg\phi & - & \neg\phi & - \\ - & - & - & - \end{bmatrix} \wedge \pi \neq \begin{bmatrix} \phi & \phi & \neg\phi & - \\ \phi & \phi & - & - \\ \phi & - & \neg\phi & - \\ - & - & - & - \end{bmatrix}$$

Condition 8 *If the difference pair $(y - x)$ is* ¬FCON*, the union-difference pairs for $(y - x)$ cannot both be* FCON*.*

$$\pi \neq \begin{bmatrix} \phi & \phi & - & \phi \\ \phi & \phi & - & - \\ - & - & - & - \\ \neg\phi & - & - & \neg\phi \end{bmatrix} \wedge \pi \neq \begin{bmatrix} \phi & \phi & - & \neg\phi \\ \phi & \phi & - & - \\ - & - & - & - \\ \phi & - & - & \neg\phi \end{bmatrix}$$

Condition 9 *All the pairs cannot be* FCON *simultaneously.*

$$\pi \neq \begin{bmatrix} \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi \\ \phi & \phi & \phi & \phi \end{bmatrix}$$

Existing Instantaneous Relation Matrices

The valid instantaneous relation matrices can be determined by successively applying the above conditions and cancelling the corresponding non-existing relations from the set of all 2^{16} relations. Four relations remain for two histories in transition through an instantaneous relationship[10] . This has been verified by finding their geometric interpretation.

Prop 1 *The possible transition matrices for relations which hold instantaneously between two histories are (i)* $\mathsf{IM}^{\pi^{\mathsf{eq}}}(\mathsf{eq}, x, y, z_1, z_2)$*, (ii)* $\mathsf{IM}^{\pi^{\mathsf{ec}}}(\mathsf{ec}, x, y, z_1, z_2)$*, (iii)* $\mathsf{IM}^{\pi^{\mathsf{tpp}}}(\mathsf{tpp}, x, y, z_1, z_2)$*, (iv)* $\mathsf{IM}^{\pi^{\mathsf{tppi}}}(\mathsf{tppi}, x, y, z_1, z_2)$*.*
The corresponding possible values for π are displayed below in the same order:

$$(i)\ \pi^{\mathsf{eq}} = \begin{bmatrix} \phi & \phi & - & - \\ \phi & \phi & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$(ii)\ \pi^{\mathsf{ec}} = \begin{bmatrix} \phi & - & - & - \\ - & - & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

[10][Galton, 2000] terms these "states of position" as compared to the other relations which cannot hold instantaneously ("states of motion").

$$(iii)\ \pi^{\mathsf{tpp}} = \begin{bmatrix} \phi & \phi & - & \phi \\ \phi & \phi & - & - \\ - & - & - & - \\ \phi & - & - & \phi \end{bmatrix}$$

$$(iv)\ \pi^{\mathsf{tppi}} = \begin{bmatrix} \phi & \phi & \phi & - \\ \phi & \phi & - & - \\ \phi & - & \phi & - \\ - & - & - & - \end{bmatrix}$$

where $- = \neg\phi$.

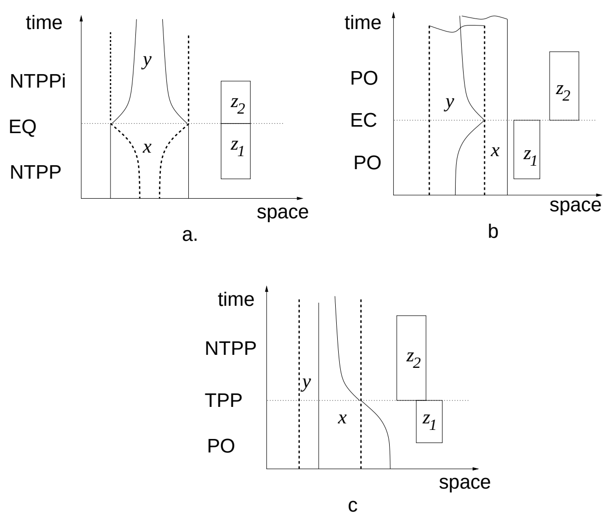

Figure 12: Instantaneous Relations possible between two histories x and y.

Fig. 12 show the relations that can hold instantaneously between two histories x and y corresponding to the four subcases of Prop 1.

It might be wondered why it takes a matrix involving 16 conditions over eight parts of x and y to identify the instantaneous relations and the conditions under which they can hold. It might turn out that it is in fact possible to characterise the conditions using a smaller set of conditions. However our intention was not to prejudge the final outcome, but rather to exhaustively analyse the relationships between the various parts of x and y without any preconception as to which relations could in fact be instantaneous and 'discover' the set analytically from the complete space of possible matrices. By conducting the analysis in this way we can have confidence that we have not missed a condition (an adhoc style of analysis might easily identify a sufficient condition but might not identify all sufficient conditions). This analysis is rather in the style of the 4- and 9-intersection model of Egenhofer [Egenhofer and Franzosa, 1991; Egenhofer and Herring, 1994; Egenhofer and Franzosa, 1995] where from a 2 x 2 and 3 x 3 matrix which determine whether various topological parts of two regions share points or not, then by imposing a variety of conditions (such as regularity or one pieceness), the 2^4 or 2^9 possibilities are whittled down to just eight possibilities (corresponding to the RCC-8 relations).

3.2 Transition Operators

We can now define three transition operators. The first two operators assume that the initial and/or the final relations hold over intervals[11] and differ as to which of the two relations hold at the dividing instant. The third is for histories undergoing a transition between two relations with an instantaneous relation holding in between.

Trans-To

A transition for two histories x and y from relation R1 over z_1 to relation R2 over z_2 occurs just in case z_1 meets z_2 and R1 holds over every initial part of the histories restricted to z_1 and R2 holds over x and y restricted to z_2.

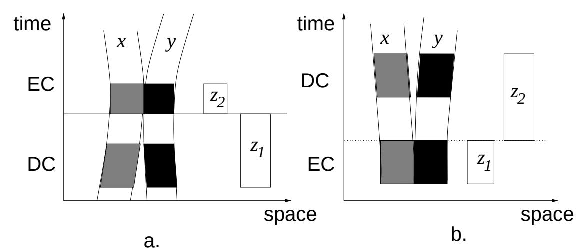

Figure 13: **a**. $\mathsf{TransTo}$ and **b**. $\mathsf{TransFrom}$ for space-time histories x and y at end of z

D25. $\mathsf{TransTo}(r1, r2, x, y, z_1, z_2) \equiv_{def} \forall u, v[[\mathsf{IP}u\frac{x}{z_1} \wedge \mathsf{IP}v\frac{y}{z_1}] \rightarrow \mathsf{rcc_{sp}}(r1, u, v)] \wedge z_1 \Join_{\mathsf{t}} z_2 \wedge \mathsf{rcc_{sp}}(r2, x/z_2, y/z_2)$

Trans-From

A transition for two histories x and y from relation R1 to relation R2 occurs just in case there exists an interval z_2 just after z_1 such that R1 holds over x and y restricted to z_1 and R2 holds over every final part of the histories restricted to z_2.

D26. $\mathsf{TransFrom}(r1, r2, x, y, z_1, z_2) \equiv_{def} \mathsf{rcc_{sp}}(r1, x/z_1, y/z_1) \wedge z_1 \Join_{\mathsf{t}} z_2 \wedge \forall u, v[[\mathsf{FP}u\frac{x}{z_2} \wedge \mathsf{FP}v\frac{y}{z_2}] \rightarrow \mathsf{rcc_{sp}}(r2, u, v)]$

The following proposition hold:

Prop 2 $\forall(r1, r2, x, y, z_1, z_2)[\neg\mathsf{TransFrom}(r1, r2, x, y, z_1, z_2) \vee \neg\mathsf{TransTo}(r1, r2, x, y, z_1, z_2)]$

Ins-rel

Any transition for two histories x and y with an instantaneous relation r holding in between z_1 and z_2 is related by the instantaneous matrix $\mathsf{IM}^{\pi^r}(r, x, y, z_1, z_2)$

D27. $\mathsf{InsRel}(r, x, y, z_1, z_2) \equiv_{def} [z_1 \Join_{\mathsf{t}} z_2 \wedge \mathsf{IM}^{\pi^r}(r, x, y, z_1, z_2)$

[11] Note that in the definitions below, the final two arguments to the $\mathsf{rcc_{sp}}$ predicate are always co-temporal, so these amount to just testing the spatial topology at the specified time.

For each instantaneous relation holding in between z_1 and z_2, distinct RCC relations hold before and after it. We introduce the following relation relating the three relations:

D28. $\mathsf{InsRel3}(r1, r2, r3, x, y, z_1, z_2) \equiv_{def}$
$$[\mathsf{InsRel}(r2, x, y, z_1, z_2) \wedge \forall u[\mathsf{IP}uz_1 \rightarrow \mathsf{rcc_{sp}}(r1, \tfrac{x}{u}, \tfrac{y}{u})] \wedge \forall u[\mathsf{FP}uz_2 \rightarrow \mathsf{rcc_{sp}}(r3, \tfrac{x}{u}, \tfrac{y}{u})]]$$

We can now define an *elementary transition* from an interval z_1 to an adjacent interval z_2 as being a TransTo, a TransFrom or an InsRel3; $r1$ is the relation that holds at the start of the transition, $r3$ is the relation that holds at the end of the transition, and $r2$ is the relation that holds instantaneously between z_1 and z_2:

D29. $\mathsf{EleTran}(r1, r2, r3, x, y, z_1, z_2) \equiv_{def}$
$$[\mathsf{TransTo}(r1, r2, x, y, z_1, z_2) \wedge r2 = r3] \vee [\mathsf{TransFrom}(r1, r3, x, y, z_1, z_2) \wedge r2 = r1] \vee [\mathsf{InsRel3}(r1, r2, r3, x, y, z_1, z_2)]$$

Transitions need to be continuous; therefore we add axioms A14 and A15. A14 states that for any TransTo to be followed by TransFrom the intermediate state is equivalent. Axiom A15 establishes an elementary transition during an interval z to be possible only for histories continuous over that interval.

A14. $[[\mathsf{TransTo}(r1, r2, x, y, z_1, z_2) \wedge \mathsf{TransFrom}(r3, r4, x, y, z_2, z_3)] \rightarrow r2 = r3]$

A15. $[\mathsf{EleTran}(r1, r2, r3, x, y, z_1, z_2) \rightarrow [\mathsf{StrFCONT}\frac{x}{(z_1 \cup z_2)} \wedge \mathsf{StrFCONT}\frac{y}{(z_1 \cup z_2)}]]$

4 Conceptual Neighbourhood of RCC-8

We can use the above formulation to recover the RCC-8 conceptual neighbourhood diagram (for $\mathfrak{CS}$-0). We need to show that the links not in the diagram represent inconsistent transitions, and that the links in the diagram are consistent transitions. The latter can be demonstrated at least intuitively by displaying an actual situation diagrammatically (e.g. see fig. 13)[12]. We will now sketch how one of the missing links corresponds to an inconsistent transition though we do not display a completely rigorous proof here. The others can be derived by reasoning along similar lines.

Prop 3 *Non-Existence of Elementary Transition between* DC *and* EQ.

Proof: A link between $r1$ and $r2$ in the conceptual neighbourhood diagram exists iff the following formula is consistent: $\exists(r, x, y, z_1, z_2)[\mathsf{EleTran}(r1, r2, r, x, y, z_1, z_2) \vee \mathsf{EleTran}(r, r1, r2, x, y, z_1, z_2) \vee \mathsf{EleTran}(r2, r1, r, x, y, z_1, z_2) \vee \mathsf{EleTran}(r, r2, r1, x, y, z_1, z_2)]$

In each case, by axiom A15, we must have that $\mathsf{StrFCONT}\frac{x}{(z_1 \cup z_2)} \wedge \mathsf{StrFCONT}\frac{y}{(z_1 \cup z_2)}$. From D29 we can infer one of:

c1. $\mathsf{TransTo}(\mathsf{eq}, \mathsf{dc}, x, y, z_1, z_2)$
c2. $\mathsf{TransFrom}(\mathsf{eq}, \mathsf{dc}, x, y, z_1, z_2)$
c3. $\mathsf{TransTo}(\mathsf{dc}, \mathsf{eq}, x, y, z_1, z_2)$
c4. $\mathsf{TransFrom}(\mathsf{dc}, \mathsf{eq}, x, y, z_1, z_2)$
c5. $\mathsf{InsRel3}(\mathsf{dc}, \mathsf{eq}, r, x, y, z_1, z_2)$
c6. $\mathsf{InsRel3}(r, \mathsf{eq}, \mathsf{dc}, x, y, z_1, z_2)$
c7. $\mathsf{InsRel3}(\mathsf{eq}, \mathsf{dc}, r, x, y, z_1, z_2)$
c8. $\mathsf{InsRel3}(r, \mathsf{dc}, \mathsf{eq}, x, y, z_1, z_2)$

c7 and c8 are immediately inconsistent since by Prop1 dc cannot be an instantaneously holding relation. In cases c1 and c2 consider the history x: it is EQ to y and then immediately DC – thus the history x must comprise two components since there must be an initial component spatially located where y is and then a spatially disconnected component separated from y; thus the history x is not StrCONT and thus not StrFCONT! Now consider the case c5 (a similar argument holds for c6): initially x and y are DC; then they are instantaneously EQ; there are two possibilities for the relationship between x and y during z_2: either the histories continue spatially connected to their respective parts during z_1 – but in this case there will be isolated points from x in y (or vice versa) instantaneously to make them EQ; or one or both the histories do not continue to be spatially connected to their parts during z_1, in which case by an argument similar to that made for c1 and c2, history x is not StrCONT and thus not StrFCONT. Thus every possibility leads to an inconsistency and EQ and DC cannot be neighbours in the conceptual neighbourhood transition graph. $\Box$

5 Conclusion

We have formally characterised different intuitive notions of s-t continuity. The strongest notion of s-t continuity has been formally defined and transition rules for s-t histories formulated in a pure, pointless, mereotopology. Our formulation is similar to Muller's, but avoids the flaws found by Davis and only requires a simpler mereotopology which does not have closure and interior operators. Nor does it have the explicit temporal points used by Davis. We have sketched how the formulation might be used to recover the RCC-8 conceptual neighbourhood diagram to be found in the literature. The axiomatisation of transitions under different notions of s-t continuity is part of ongoing research as is a completely formal proof of the correctness of the RCC-8 conceptual neighbourhood diagram. It should also be straightforward to recover the dominance theory of Galton which is a refinement of the conceptual neighbourhood diagram and closely corresponds to the three different kinds of transition we have identified here (Transto, Transfrom and Insrel).

[12] We recognise that this is not satisfying from a formal point of view and one really want a rigorous proof that the full theory conjoined with a sentence such as $\exists x, y, z1, z2 \mathsf{Transto}(\mathsf{dc}, \mathsf{ec}, x, y, z1, z2)$ is satisfiable.

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