Reasoning About Knowledge and Action in an Epistemic Event Calculus

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Abstract

We present a generalization of classical-logic Event Calculus that facilitates reasoning about non-binary-valued fluents in domains with non-deterministic, triggered, concurrent, and possibly conflicting actions. We show how this framework may be used as a basis for a "possible-worlds" style approach to epistemic and causal reasoning in a narrative setting. In this framework an agent may gain knowledge about both fluent values and action occurrences through sensing actions, lose knowledge via non-deterministic actions, and represent plans that include conditional actions whose conditions may be initially unknown.

1 Introduction

The Event Calculus (EC) (Kowalski and Sergot 1986; Miller and Shanahan 2002) is a well-established technique within AI for representing causal and narrative information about dynamic domains. However, compared to other action formalisms, little work has been done in developing epistemic extensions to the EC to facilitate reasoning about an agent's changing state of knowledge and the state of its environment. An exception is (Patkos and Plexousakis 2009), which develops an epistemic extension to the Discrete Event Calculus of (Mueller 2006), using a deduction-oriented rather than possible-worlds based model of knowledge. In this paper we propose an alternative epistemic EC variant, the *Epistemic Functional Event Calculus* (EFEC), that builds on a generalization of the EC of (Miller and Shanahan 2002). Differentiating characteristics of the EFEC are that (i) time can be either discrete or continuous (real-valued), (ii) it generalizes the EC to include non-binary (i.e. non-truth-valued) fluents, (iii) it uses a possible-worlds notion to model knowledge, following (Scherl and Levesque 1993) and others, (iv) it facilitates reasoning about domains with concurrent, non-deterministic, and possibly conflicting actions, (v) it enables reasoning about domains with "triggered" or "natural" actions, and can model states of knowledge about action occurrences as well as fluent values, and (vi) it is able to represent knowledge states about past and future times relative to the agent's "now" as well as the present. Section 4 gives comparison to existing related work. Diagrams and other resources to aid the reader are available online at `http://www.ucl.ac.uk/infostudies/efec`.

Notation: We use sorted classical predicate calculus with equality. All variables are universally quantified with maximum scope unless otherwise indicated.

2 Functional Event Calculus

As a foundation for our epistemic EC, we first give a generalization of the non-deterministic EC of (Miller and Shanahan 2002) to include non-binary-valued fluents, which we refer to as the FEC. This brings the EC on a par with action formalisms such as the Situation Calculus in this respect.

The FEC has a sort $\mathcal{A}$ for *actions* (variables $a, a', a_1, \ldots$), a sort $\mathcal{F}$ for fluents ($f, f', f_1, \ldots$), a sort $\mathcal{V}$ for values ($v, v', v_1, \ldots$) and a sort $\mathcal{T}$ for timepoints ($t, t', t_1, \ldots$). For this section only, the reader may assume that time is modeled as a total ordering (e.g. $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{N}$ or $\mathbb{Z}$). The key predicates and functions are $\mathit{Happens} \subseteq \mathcal{A} \times \mathcal{T}$, $\mathit{ValueOf} : \mathcal{F} \times \mathcal{T} \rightarrow \mathcal{V}$, $\mathit{CausesValue} \subseteq \mathcal{A} \times \mathcal{F} \times \mathcal{V} \times \mathcal{T}$, $\mathit{PossVal} \subseteq \mathcal{F} \times \mathcal{V}$, and $< \subseteq \mathcal{T} \times \mathcal{T}$. To describe the general relationship between these predicates it is convenient to first define two auxiliary predicates, $\mathit{ValueCaused} \subseteq \mathcal{F} \times \mathcal{V} \times \mathcal{T}$ and $\mathit{OtherValCausedBetween} \subseteq \mathcal{F} \times \mathcal{V} \times \mathcal{T} \times \mathcal{T}$. $\mathit{ValueCaused}(F, V, T)$ means that some action happens at T that gives cause for F to take value V. $\mathit{OtherValCausedBetween}(F, V, T_1, T_2)$ means that some action happens at some point in the half-open interval $[T_1, T_2)$ that gives cause for F to take a value other than V. Note that *gives cause* is a weaker notion than the standard *causes*: non-deterministic actions do not cause specific predictable effects. E.g. (example 2.1.A), rolling a die *gives cause* for each number to show, but we cannot predict *which* number will show.

$$\mathit{ValueCaused}(f, v, t) \stackrel{\text{def}}{\equiv} \exists a[\mathit{Happens}(a, t) \wedge \mathit{CausesValue}(a, f, v, t)]. \quad \text{(FEC1)}$$

$$\mathit{OtherValCausedBetween}(f, v, t_1, t_2) \stackrel{\text{def}}{\equiv} \exists t, v'[\mathit{ValueCaused}(f, v', t) \wedge t_1 \leq t < t_2 \wedge v \neq v']. \quad \text{(FEC2)}$$

The notions of cause, effect, and inertia are captured in two FEC axioms. (FEC3) states that a fluent has a particular value at a particular time if either (i) it already had that value at an earlier time or (ii) was given cause to take that value from an earlier time, and in the meantime (including that earlier time) nothing has happened that might give cause for it to take an alternative value. Conversely, (FEC4) states that fluent f cannot have value v at time t_2 if its most recent causal influences prior to t_2 do not include a cause for v. Finally, (FEC5) additionally constrains each fluent's value to be at all times among the set of values defined by *PossVal*:

$$\mathit{ValueOf}(f,t_2)=v \leftarrow \quad \text{(FEC3)}$$
$$[(\mathit{ValueOf}(f,t_1)=v \vee \mathit{ValueCaused}(f,v,t_1)) \wedge t_1<t_2 \wedge \neg \mathit{OtherValCausedBetween}(f,v,t_1,t_2)].$$

$$\mathit{ValueOf}(f,t_2)\neq v \leftarrow \quad \text{(FEC4)}$$
$$[t_1<t_2 \wedge \mathit{OtherValCausedBetween}(f,v,t_1,t_2) \wedge \neg\exists t(t_1\leq t<t_2 \wedge \mathit{ValueCaused}(f,v,t))].$$

$$\mathit{ValueOf}(f,t)=v \rightarrow \mathit{PossVal}(f,v). \quad \text{(FEC5)}$$

Definitions of the predicates *Happens*, *CausesValue* and *PossVal* are given in the domain-dependent part of the theory (or partially defined and then minimised to address the Frame Problem and related issues). The reader may note that if $\mathit{PossVal}(f,v) \equiv [v=\mathit{True} \vee v=\mathit{False}]$ for all fluents, axioms (FEC1)-(FEC5) are effectively equivalent to axioms (EC1)-(EC6) from (Miller and Shanahan 2002).

Example FEC Domain Representations

A. Rolling a Die. This illustrates that we can capture non-determinism by expressing with *CausesValue* that an action can give cause for more than one value for a given fluent. The effect of rolling a 6-sided die twice (at times 10 and 20) after it initially shows "2" is described as follows:

$$f=\mathit{DieFaceShowing} \wedge a=\mathit{RollDie}. \quad \text{(D1)}$$
$$v=1\vee v=2\vee v=3\vee v=4\vee v=5\vee v=6. \quad \text{(D2)}$$
$$\mathit{PossVal}(\mathit{DieFaceShowing},v). \quad \text{(D3)}$$
$$\mathit{CausesValue}(\mathit{RollDie},\mathit{DieFaceShowing},v,t). \quad \text{(D4)}$$
$$\mathit{ValueOf}(\mathit{DieFaceShowing},0)=2. \quad \text{(D5)}$$
$$\mathit{Happens}(\mathit{RollDie},t) \equiv [t=10\vee t=20]. \quad \text{(D6)}$$

(D1) and (D2) are domain closure axioms. (D4) expresses that rolling a die gives cause for each of its six faces to show uppermost. (In a more complex domain (D4) might be expressed as the circumscription of a collection of *CausesValue* clauses.) (D5) and (D6) express *narrative information* about the domain. Again, (D6) could alternatively be expressed as the circumscription of the clauses $\mathit{Happens}(\mathit{RollDie},10)$ and $\mathit{Happens}(\mathit{RollDie},20)$. If time is represented by $\mathbb{R}_{\geq 0}$, this example gives rise to 6^2 models. In each model the die has face "2" showing throughout the interval $[0,10]$, and an arbitrary one of its six faces showing throughout each of the intervals $(10,20]$ and $(20,+\infty)$.

B. Pushing and Pulling a Gate. This example shows that concurrent occurrence of conflicting actions can be treated as non-determinism. Provided a gate is unbroken, pushing gives cause for it to shut and pulling gives cause for it to open. The gate is both pushed and pulled at time 2:

$$f=\mathit{GateStatus} \wedge (a=\mathit{Push}\vee a=\mathit{Pull}). \quad \text{(G1)}$$
$$v=\mathit{Open}\vee v=\mathit{Shut}\vee v=\mathit{Broken}. \quad \text{(G2)}$$
$$\mathit{Open}\neq\mathit{Shut}\neq\mathit{Broken}. \quad \text{(G3)}$$
$$\mathit{PossVal}(\mathit{GateStatus},v). \quad \text{(G4)}$$
$$\mathit{CausesValue}(\mathit{Push},\mathit{GateStatus},v,t) \equiv [v=\mathit{Shut}\wedge \mathit{ValueOf}(\mathit{GateStatus},t)\neq\mathit{Broken}]. \quad \text{(G5)}$$
$$\mathit{CausesValue}(\mathit{Pull},\mathit{GateStatus},v,t) \equiv [v=\mathit{Open}\wedge \mathit{ValueOf}(\mathit{GateStatus},t)\neq\mathit{Broken}]. \quad \text{(G6)}$$
$$\mathit{ValueOf}(\mathit{GateStatus},0)\neq\mathit{Broken}. \quad \text{(G7)}$$
$$\mathit{Happens}(a,t) \equiv [(a=\mathit{Pull}\vee a=\mathit{Push})\wedge t=2]. \quad \text{(G8)}$$

Taking the sort $\mathcal{T}$ to be $\mathbb{R}$, this FEC theory has four models, in which *GateStatus* takes one of the values *Open* and *Shut* throughout each of the intervals $(-\infty,2]$, and $(2,+\infty)$. Axiom (FEC4) ensures that there are no models in which *GateStatus* takes the value *Broken* during $(2,+\infty)$. We could remove the non-determinism for this domain, and state that simultaneously pulling and pushing breaks the gate, by replacing (G5) and (G6) with the following:

$$\mathit{CausesValue}(\mathit{Push},\mathit{GateStatus},v,t) \equiv \quad \text{(G5}'\text{)}$$
$$[(\neg\mathit{Happens}(\mathit{Pull},t)\wedge \mathit{ValueOf}(\mathit{GateStatus},t)\neq\mathit{Broken} \wedge v=\mathit{Shut}) \vee (\mathit{Happens}(\mathit{Pull},t)\wedge v=\mathit{Broken})].$$
$$\mathit{CausesValue}(\mathit{Pull},\mathit{GateStatus},v,t) \equiv \quad \text{(G6}'\text{)}$$
$$[(\neg\mathit{Happens}(\mathit{Push},t)\wedge \mathit{ValueOf}(\mathit{GateStatus},t)\neq\mathit{Broken} \wedge v=\mathit{Open}) \vee (\mathit{Happens}(\mathit{Push},t)\wedge v=\mathit{Broken})].$$

3 Epistemic Reasoning

A Motivating Example – the Shopping Outlet: *To obtain an item from a catalogue shopping outlet, a customer first purchases the item using a terminal. This causes the customer to be (non-deterministically) assigned to one of three collection points color-coded "red", "blue" and "green". The customer can ascertain which collection point she has been assigned to from a display, and collect the item accordingly. To discourage unnecessary queuing, an attempt to collect the item from the wrong collection point cancels the purchase. Additionally, customers assigned to "red" receive a free gift on successful item collection.* This scenario can be regarded as having an action (purchasing the item) for which one effect is non-deterministic, a "sensing" or knowledge-producing action (ascertaining the collection point), conditional actions (collect from a particular collection point *if* assigned there), and a conditionally "triggered" action (giving a gift at the red collection point). The example is pertinent because it is impossible for the customer to obtain the item for certain without the sensing action (assuming she cannot collect from all three collection points simultaneously).

Other than "epistemic fluents" (described later) our representation of this example will use the three fluents *CollectionPoint*, *Bought* and *Collected*. *CollectionPoint* has possible values $\{\mathit{Red},\mathit{Blue},\mathit{Green}\}$, *Bought* and *Collected* are truth-valued. Other than sensing actions (also described later) we will use the five actions *Purchase*, *CollectFromRed*, *CollectFromBlue*, *CollectFromGreen* and *GiveFreeGift*. *GiveFreeGift* is a "triggered" action performed by the environment automatically in the circumstances described. The UNA axioms for values and definition of *PossVal* for the non-epistemic fluents are:

$$\mathit{PossVal}(\mathit{Bought},v) \equiv [v=\mathit{True}\vee v=\mathit{False}]. \quad \text{(S1)}$$
$$\mathit{PossVal}(\mathit{CollectionPoint},v) \equiv [v=\mathit{Red}\vee v=\mathit{Blue}\vee v=\mathit{Green}]. \quad \text{(S2)}$$
$$\mathit{PossVal}(\mathit{Collected},v) \equiv [v=\mathit{True}\vee v=\mathit{False}]. \quad \text{(S3)}$$
$$\mathit{Red}\neq\mathit{Blue}\neq\mathit{Green}\neq\mathit{True}\neq\mathit{False}. \quad \text{(S4)}$$

To fully represent this domain, we need a time structure, predicates and axioms to facilitate reasoning about knowledge.

EFEC Knowledge Axioms

Our approach to epistemic reasoning is inspired by the "possible-worlds" approaches of (Scherl and Levesque 1993; Moore 1980; Fagin et al. 1995) and others, but has some ontological differences necessitated by the fact that the EC is a narrative framework, and typically uses linear (not branching) time. Intuitively, each model of an *epistemic* FEC (EFEC) theory contains a number of parallel "possible time lines" and a notion of accessibility (modeled via a collection of "epistemic fluents") between these. The time lines accessible to an agent at any given moment are those that contain a narrative (fluent values and action occurrences) compatible with the agent's current state of knowledge. In other words, at a given instant the agent "knows" whatever holds in every narrative accessible at that instant. Sensing actions terminate the accessibility of time lines whose narratives do not match the sensed information. Once terminated, there is no mechanism for re-initiating accessibility. Loss of knowledge due to non-deterministic action occurrences is instead captured by ensuring (in every model) that, for every possible history of previous fluent values and action occurrences up to the given point of non-determinism, there is a sufficient number of identical, duplicate accessible narratives to ensure that at least one such narrative is available to be extended to reflect each possible non-deterministic outcome.

To represent time as a system of parallel time lines as described above, we add two new sorts to the logic, a sort $\mathcal{W}$ for *worlds* that are understood as identifiers for possible time lines (variables $w, w', w_1, ...$) and a sort $\mathcal{I}$ for *instants* (variables $i, i', i_1, ...$). The constant W_a of sort $\mathcal{W}$ signifies the "actual world", and $\prec\ \subseteq \mathcal{I} \times \mathcal{I}$ is a partial (possibly total) ordering over $\mathcal{I}$. The function $\langle\ \rangle : \mathcal{W} \times \mathcal{I} \rightarrow \mathcal{T}$ maps world/instant pairs to time points, so that time point $\langle W, I\rangle$ represents "instant I in possible world W". We write $\langle I\rangle$ as shorthand for $\langle W_a, I\rangle$. Axioms for the time structure are:

$$\forall t \exists w, i.(t = \langle w, i\rangle). \quad \text{(EFEC1)}$$

$$\langle w, i\rangle = \langle w', i'\rangle \equiv (w = w' \wedge i = i'). \quad \text{(EFEC2)}$$

$$\langle w, i\rangle < \langle w', i'\rangle \equiv (w = w' \wedge i \prec i'). \quad \text{(EFEC3)}$$

$\leq$, $>$ and $\geq$ have their usual meanings in terms of $=$ and $<$. For ease of exposition, in this paper we assume $\mathcal{I}$ is interpreted as $\mathbb{R}_{\geq 0}$ with usual ordering $\prec$ unless otherwise stated.

To specify the dynamic accessibility relation between possible worlds, we adapt Scherl and Levesque's notion of *epistemic fluents*, introducing a function $K : \mathcal{W} \rightarrow \mathcal{F}$. The epistemic fluent $K(W)$ represents the "accessibility" property of W, so that $\mathit{ValueOf}(K(W), \langle W', I\rangle) = \mathit{true}$ means that "W is accessible from W' at instant I". K is a truth-valued fluent, and, since we are modeling knowledge, the relationship it represents between worlds is reflexive, symmetric and transitive (i.e. an equivalence relation):

$$\mathit{PossVal}(K(w), v) \equiv [v = \mathit{True} \vee v = \mathit{False}]. \quad \text{(EFEC4)}$$

$$\mathit{ValueOf}(K(w), \langle w, i\rangle) = \mathit{True}. \quad \text{(EFEC5)}$$

$$\mathit{ValueOf}(K(w), \langle w', i\rangle) = \mathit{True} \equiv \mathit{ValueOf}(K(w'), \langle w, i\rangle) = \mathit{True}. \quad \text{(EFEC6)}$$

$$\mathit{ValueOf}(K(w_3), \langle w_1, i\rangle) = \mathit{True} \leftarrow [\mathit{ValueOf}(K(w_2), \langle w_1, i\rangle) = \mathit{True} \wedge \mathit{ValueOf}(K(w_3), \langle w_2, i\rangle) = \mathit{True}]. \quad \text{(EFEC7)}$$

For simplicity, we assume that each fluent F in the domain can be sensed via an action $\mathit{Sense}(F)$ (although nothing in our approach dictates that this must be so[1]). Sensing F terminates the accessibility of possible worlds in which the value of F is different from that in the agent's world:

$$\mathit{CausesValue}(\mathit{Sense}(f), K(w'), \mathit{False}, \langle w, i\rangle) \leftarrow \mathit{ValueOf}(f, \langle w, i\rangle) \neq \mathit{ValueOf}(f, \langle w', i\rangle). \quad \text{(EFEC8)}$$

EFEC includes six knowledge predicates regarding actions and non-epistemic fluents, whose arity and argument sorts can be seen from the following list of intended meanings:

- $\mathit{KnowsValueIsNot}(\langle W, I\rangle, F, I', V)$ - When in world W at instant I, the agent knows that the value of fluent F at instant I' was not / is not / will not be V.
- $\mathit{KnowsValueIs}(\langle W, I\rangle, F, I', V)$ - When in W at instant I, the agent knows that the value of F at I' was / is / will be V.
- $\mathit{KnowsValue}(\langle W, I\rangle, F, I')$ - When in W at instant I, the agent knows the value of F at instant I'.
- $\mathit{KnowsHappens}(\langle W, I\rangle, A, I')$ - When in W at instant I, the agent knows that action A has happened / happens / will happen at instant I'.
- $\mathit{KnowsNotHappens}(\langle W, I\rangle, A, I')$ - In W at instant I, the agent knows that A has not / does not / will not happen at I'.
- $\mathit{KnowsIfHappens}(\langle W, I\rangle, A, I')$ - When in W at instant I, the agent knows whether or not A has happened / happens / will happen at I'.

The corresponding definitional axioms are: [2]

$$\mathit{KnowsValueIsNot}(\langle w, i\rangle, f, i', v) \equiv \forall w' (f \neq K(w') \wedge [\mathit{ValueOf}(K(w'), \langle w, i\rangle) = \mathit{True} \rightarrow \mathit{ValueOf}(f, \langle w', i'\rangle) \neq v]). \quad \text{(EFEC9)}$$

$$\mathit{KnowsValueIs}(\langle w, i\rangle, f, i', v) \equiv \forall v' [(\mathit{PossVal}(f, v') \wedge v' \neq v) \rightarrow \mathit{KnowsValueIsNot}(\langle w, i\rangle, f, i', v')]. \quad \text{(EFEC10)}$$

$$\mathit{KnowsValue}(t, f, i') \equiv \exists v. \mathit{KnowsValueIs}(t, f, i', v). \quad \text{(EFEC11)}$$

$$\mathit{KnowsHappens}(\langle w, i\rangle, a, i') \equiv \forall w' [\mathit{ValueOf}(K(w'), \langle w, i\rangle) = \mathit{True} \rightarrow \mathit{Happens}(a, \langle w', i'\rangle)]. \quad \text{(EFEC12)}$$

$$\mathit{KnowsNotHappens}(\langle w, i\rangle, a, i') \equiv \forall w' [\mathit{ValueOf}(K(w'), \langle w, i\rangle) = \mathit{True} \rightarrow \neg \mathit{Happens}(a, \langle w', i'\rangle)]. \quad \text{(EFEC13)}$$

$$\mathit{KnowsIfHappens}(t, a, i') \equiv [\mathit{KnowsHappens}(t, a, i') \vee \mathit{KnowsNotHappens}(t, a, i')]. \quad \text{(EFEC14)}$$

Representing Action Occurrences

To represent the class of domains exemplified by the shopping outlet scenario, we need to be able to represent three kinds of action occurrence. First, the agent may

[1] We could for example specify when an action a senses a fluent f using a predicate *Senses* along with a general rule such as $\mathit{CausesValue}(a, K(w'), \mathit{False}, \langle w, i\rangle) \leftarrow [\mathit{Senses}(a, f, \langle w, i\rangle) \wedge \mathit{ValueOf}(f, \langle w, i\rangle) \neq \mathit{ValueOf}(f, \langle w', i\rangle)]$.

[2] KnowValueIs (KVI) is defined here in terms of the arguably more basic KnowValueIsNot (KVIN). KVIN is particularly useful for expressing complete ignorance (i.e., for all possible values *v* of a fluent, one does not know that the fluent does not have value *v*). But note that KVI could instead be defined directly in terms of possible worlds; in that case, the current formulation of (EFEC10) could be derived as a corollary of (EFEC9) and the direct definition of KVI.

perform an action unconditionally (e.g. purchase an item). Second, the agent may perform an action if (and only if) it knows a particular condition holds (e.g. collect from the red collection point if assigned there). Third, an action might be automatically triggered in the environment (e.g. the giving of a free gift on item collection). Accordingly, we introduce three occurrence predicates, $Perform \subseteq \mathcal{A} \times \mathcal{I}$, $PerformIfValueKnownIs \subseteq \mathcal{A} \times \mathcal{F} \times \mathcal{V} \times \mathcal{I}$ and $Triggered \subseteq \mathcal{A} \times \mathcal{T}$ and relate them to *Happens* as follows.

$$Perform(a,i) \;\rightarrow\; Happens(a,\langle w,i\rangle). \tag{EFEC15}$$

$$PerformIfValueKnownIs(a,f,v,i) \;\rightarrow \; [Happens(a,\langle w,i\rangle) \equiv KnowsValueIs(\langle w,i\rangle,f,i,v)]. \tag{EFEC16}$$

$$Triggered(a,t) \;\rightarrow\; Happens(a,t). \tag{EFEC17}$$

Definitions of these predicates are given in the domain-dependent theory, or, in the case of the "*Perform*..." predicates, generated via a planning process. The "*Perform*..." predicates have last argument of sort $\mathcal{I}$ (rather than $\mathcal{T}$) because the occurrences they refer to are by assumption under the control of the agent. In contrast the conditions under which triggered actions occur may or may not be known at particular times.

Rather than minimising *Happens* directly, we state that all occurrences of actions (at any instant in any possible world) are accounted for by "*Perform*..." or *Triggered*:

$$Happens(a,\langle w,i\rangle) \;\rightarrow\; [\exists f,v.PerformIfValueKnownIs(a,f,v,i) \vee Perform(a,i) \;\vee\; Triggered(a,\langle w,i\rangle)]. \tag{EFEC18}$$

An Axiomatization of the Shopping Example

The domain-dependent shopping outlet axioms are as follows. In addition to (S1)–(S4) we have domain closure and uniqueness-of-names axioms for actions and fluents:

$$f = CollectionPoint \;\vee\; f = Bought \vee f = Collected \;\vee\; \exists w.f = K(w). \tag{S5}$$

$$CollectionPoint \neq Bought \neq Collected \neq K(w) \wedge [K(w) = K(w') \rightarrow w = w']. \tag{S6}$$

$$a = Purchase \vee a = CollectFromRed \vee a = CollectFromBlue \vee a = CollectFromGreen \vee a = GiveFreeGift \;\vee\; \exists f.a = Sense(f). \tag{S7}$$

$$Purchase \neq GiveFreeGift \neq CollectFromRed \neq CollectFromBlue \neq CollectFromGreen \neq Sense(f) \wedge [Sense(f) = Sense(f') \rightarrow f = f']. \tag{S8}$$

We assume sort $\mathcal{I}$ is interpreted as $\mathbb{R}_{\geq 0}$. We express knowledge about instant 0 in terms of *KnowsValueIsNot* (which gives more expressivity than *KnowsValueIs*), and completely specify *KnowsValueIsNot* at instant 0 (recall from Section 3 that $\langle 0\rangle$ is shorthand for $\langle W_a, 0\rangle$):

$$KnowsValueIsNot(\langle 0\rangle,f,0,v) \;\equiv\; [(f = Bought \;\vee\; f = Collected) \;\wedge\; v = True]. \tag{S9}$$

Causal information about the domain is captured by a collection of clauses such as

$$CausesValue(CollectFromRed, Collected, True, t) \leftarrow [ValueOf(CollectionPoint,t) = Red \wedge ValueOf(Bought,t) = True].$$

and the frame problem is addressed by circumscribing this collection together with (EFEC8) to give

$$\begin{aligned} CausesValue(a,f,v,t) \;\equiv\; & [(a = Purchase \wedge f = CollectionPoint \wedge PossVal(CollectionPoint,v)) \\ & \vee (a = Purchase \wedge f = Bought \wedge v = True) \\ & \vee (a = CollectFromRed \wedge f = Collected \wedge v = True \wedge ValueOf(CollectionPoint,t) = Red \wedge ValueOf(Bought,t) = True) \\ & \vee (a = CollectFromRed \wedge f = Bought \wedge v = False \wedge ValueOf(CollectionPoint,t) \neq Red) \\ & \vee (a = CollectFromBlue \wedge f = Collected \wedge v = True \wedge ValueOf(CollectionPoint,t) = Blue \wedge ValueOf(Bought,t) = True) \\ & \vee (a = CollectFromBlue \wedge f = Bought \wedge v = False \wedge ValueOf(CollectionPoint,t) \neq Blue) \\ & \vee (a = CollectFromGreen \wedge f = Collected \wedge v = True \wedge ValueOf(CollectionPoint,t) = Green \wedge ValueOf(Bought,t) = True) \\ & \vee (a = CollectFromGreen \wedge f = Bought \wedge v = False \wedge ValueOf(CollectionPoint,t) \neq Green) \\ & \vee \exists f',w,w',i(a = Sense(f') \wedge f = K(w') \wedge v = False \;\wedge\; t = \langle w,i\rangle \;\wedge \\ & \quad ValueOf(f',\langle w,i\rangle) \neq ValueOf(f',\langle w',i\rangle))]. \end{aligned} \tag{S10}$$

Various EC mechanisms compatible with the framework described here have been developed to represent triggered actions (see e.g. (Miller and Shanahan 2002)). For triggering a free gift in the shopping example, the following is sufficient:

$$Triggered(a,t) \;\equiv\; [ValueOf(Bought,t) = True \wedge Happens(CollectFromRed,t) \wedge a = GiveFreeGift \wedge ValueOf(CollectionPoint,t) = Red]. \tag{S11}$$

Finally, actions the agent has done or (conditionally) intends to do can be represented by definitions of *Perform* and *PerformIfValueKnownIs*. For example, a plan for obtaining the item by instant 4 might be described by:

$$Perform(a,i) \;\equiv\; [(a = Purchase \wedge i = 1) \vee (a = Sense(CollectionPoint) \wedge i = 2)]. \tag{S12}$$

$$\begin{aligned} PerformIfValueKnownIs(a,f,v,i) \;\equiv\; & [(a = CollectFromRed \wedge f = CollectionPoint \wedge v = Red \wedge i = 3) \\ & \vee (a = CollectFromBlue \wedge f = CollectionPoint \wedge v = Blue \wedge i = 3) \\ & \vee (a = CollectFromGreen \wedge f = CollectionPoint \wedge v = Green \wedge i = 3)]. \end{aligned} \tag{S13}$$

Existence of Possible Worlds

The domain-independent EFEC axiomatization is not yet complete. We need to ensure there is a sufficiently large collection of accessible possible worlds in each model to adequately represent lack of knowledge, both about what holds at the initial instant and about what holds after the occurrence of a set of simultaneous non-deterministic actions.

For example, if we have no knowledge about the initial values of N truth-valued fluents, our axiomatization should ensure there are at least 2^N initially accessible worlds, one for each possible N-combination of truth values. If we were to allow models with less than 2^N such worlds, then in these models some sensing actions would (by terminating accessibility – see (EFEC8)) give unwarranted knowledge about the values of fluents other than that being sensed.

To eliminate such models in the general case we first axiomatize the notion that two worlds differ at instant 0 in respect of non-epistemic fluents by at most one such fluent, by defining a predicate *InitiallyDifferAtMostBy* $\subseteq \mathcal{W}\times\mathcal{W}\times\mathcal{F}$:

$$\mathit{InitiallyDifferAtMostBy}(w_1, w_2, f) \equiv \quad \text{(EFEC19)}$$
$$\forall f'[(f' \neq f \ \wedge\ \neg\exists w'.f' = K(w')) \ \rightarrow$$
$$\mathit{ValueOf}(f', \langle w_1, 0\rangle) = \mathit{ValueOf}(f', \langle w_2, 0\rangle)].$$

Domain descriptions will typically include at least a partial specification for *KnowsValueIsNot* at instant 0 (see e.g. (S9)). Axiom (EFEC20) states that for every value v not known not to be the initial value of some non-epistemic fluent f, and for every initially accessible world w, we can find an initially accessible world w' in which f has initial value v and which is initially identical to w in all other respects:

$$[\neg\exists w'.f = K(w') \ \wedge\ \mathit{PossVal}(f, v) \ \wedge \quad \text{(EFEC20)}$$
$$\neg\mathit{KnowsValueIsNot}(\langle 0\rangle, f, 0, v) \ \wedge \mathit{ValueOf}(K(w), \langle 0\rangle) = \mathit{True}]$$
$$\rightarrow \exists w'[\mathit{ValueOf}(K(w'), \langle 0\rangle) = \mathit{True} \ \wedge$$
$$\mathit{ValueOf}(f, \langle w', 0\rangle) = v \ \wedge \mathit{InitiallyDifferAtMostBy}(w, w', f)].$$

For domains with a finite number of fluents, it is possible to show that any combination of fluent values, each of which are not known not to hold, corresponds to an initially accessible world, by repeated application of (EFEC20).

To guarantee the existence of a sufficient number of worlds accessible after the occurrence of a set of simultaneous non-deterministic actions, we first need to be able to identify time periods "immediately after" such occurrences. To do this we include the function $\mathit{Next} : \mathcal{T} \rightarrow \mathcal{T}$ from (Miller and Shanahan 1996). For time point T axioms (EFEC21)–(EFEC23) constrain $\mathit{Next}(T)$ as follows. If T is before the last action occurrence in T's timeline, then $\mathit{Next}(T)$ is the point of the next action occurrence (or simultaneous occurrences) after T. Otherwise, $Next(T)$ is any arbitrary time point after T.

$$t < \mathit{Next}(t). \quad \text{(EFEC21)}$$
$$[t < t_1 \ \wedge\ t_1 < \mathit{Next}(t)] \ \rightarrow\ \neg\mathit{Happens}(a, t_1). \quad \text{(EFEC22)}$$
$$[\mathit{Happens}(a_1, t_1) \wedge t < t_1] \rightarrow \exists a.\mathit{Happens}(a, \mathit{Next}(t)). \quad \text{(EFEC23)}$$

(FEC3) therefore guarantees that for any T, values of fluents remain unchanged in the half-open interval $(T, \mathit{Next}(T)]$. In particular, if actions occur at T then the immediate effects of those actions remain apparent throughout $(T, \mathit{Next}(T)]$.

Two other predicates are needed, *DifferAfterAtMostBy* $\subseteq \mathcal{W} \times \mathcal{W} \times \mathcal{I} \times \mathcal{F}$ and *EqualUpTo* $\subseteq \mathcal{W} \times \mathcal{W} \times \mathcal{I}$. $\mathit{DifferAfterAtMostBy}(W_1, W_2, I, F)$ means that in the periods immediately following instant I on each of the timelines W_1 and W_2 – i.e. in the half open intervals $(\langle W_1, I\rangle, \mathit{Next}(\langle W_1, I\rangle)]$ and $(\langle W_2, I\rangle, \mathit{Next}(\langle W_2, I\rangle)]$ – all non-epistemic fluents except possibly F take the same value. $\mathit{EqualUpTo}(W_1, W_2, I)$ means that in the period from 0 up to and including I on each of the timelines W_1 and W_2 – i.e. in the intervals $[\langle W_1, 0\rangle, \langle W_1, I\rangle]$ and $[\langle W_2, 0\rangle, \langle W_2, I\rangle]$ – all non-epistemic fluents take the same value:

$$\mathit{DifferAfterAtMostBy}(w_1, w_2, i, f) \equiv \quad \text{(EFEC24)}$$
$$\forall f'[(f' \neq f \ \wedge\ \neg\exists w'.f' = K(w')) \ \rightarrow$$
$$\mathit{ValueOf}(f', \mathit{Next}(\langle w_1, i\rangle)) = \mathit{ValueOf}(f', \mathit{Next}(\langle w_2, i\rangle))].$$
$$\mathit{EqualUpTo}(w_1, w_2, i) \equiv \quad \text{(EFEC25)}$$
$$\forall f, i'[(i' \preceq i \ \wedge\ \neg\exists w'.f = K(w')) \ \rightarrow$$
$$\mathit{ValueOf}(f, \langle w_1, i'\rangle) = \mathit{ValueOf}(f', \langle w_2, i'\rangle)].$$

Axiom (EFEC26) is the counterpart of (EFEC20) for periods immediately following (possibly non-deterministic) action occurrences. It states that if a non-epistemic fluent f is given cause to have value v at instant i in the accessible world w, then there exists another accessible world w' identical to w up to $\mathit{Next}(\langle w, i\rangle)$ except that at $\mathit{Next}(\langle w', i\rangle)$ f has value v (whereas f may or may not have value v at $\mathit{Next}(\langle w, i\rangle)$):

$$[\mathit{ValueCaused}(f, v, \langle w, i\rangle) \ \wedge\ \neg\exists w_1.f = K(w_1) \quad \text{(EFEC26)}$$
$$\wedge\ \mathit{ValueOf}(K(w), \mathit{Next}(\langle i\rangle)) = \mathit{True}]$$
$$\rightarrow \exists w'[\mathit{ValueOf}(K(w'), \mathit{Next}(\langle i\rangle)) = \mathit{True}$$
$$\wedge\ \mathit{ValueOf}(f, \mathit{Next}(\langle w', i\rangle)) = v \ \wedge\ \mathit{EqualUpTo}(w, w', i)$$
$$\wedge\ \mathit{DifferAfterAtMostBy}(w, w', i, f)].$$

Note that these axioms rest on the assumption that all pairs of fluents are orthogonal (i.e., there are no state constraints).

Example Inferences

EFEC allows us to describe *epistemically feasible* plans. For example, the goal of obtaining an item at instant 4 might be expressed as $\mathit{Goal}_S \equiv \mathit{KnowsValueIs}(\langle 0\rangle, \mathit{Collected}, 4, \mathit{True})$ and a plan to achieve this as $\mathit{Plan}_S \equiv$ [(S12) ∧ (S13)]. Taking an abductive view of planning, Proposition 1 below shows that: (FEC1) ∧ ... ∧ (FEC5) ∧ (EFEC1) ∧ ... ∧ (EFEC26) ∧ (S1) ∧ ... ∧ (S11) ∧ $\mathit{Plan}_S \models \mathit{Goal}_S$, or equivalently (under a deductive view of planning) (FEC1) ∧ ... ∧ (FEC5) ∧ (EFEC1) ∧ ... ∧ (EFEC26) ∧ (S1) ∧ ... ∧ (S11) $\models \mathit{Plan}_S \rightarrow \mathit{Goal}_S$. Proposition 1 also shows that the agent is able to infer and preserve knowledge about instances that have passed, both regarding action occurrences (in particular triggered occurrences, e.g. $\mathit{KnowsIfHappens}(\langle 5\rangle, \mathit{GiveFreeGift}, 3)$) and fluent values (e.g. $\mathit{KnowsValue}(\langle 3\rangle, \mathit{CollectionPoint}, 2)$). Consequently, the agent can plan to discover currently unknown facts about past times.

Proposition 1. The theory (FEC1)∧...∧(FEC5)∧(EFEC1)∧ ... ∧ (EFEC26) ∧ (S1) ∧ ... ∧ (S13) entails the sentences:

$$\mathit{KnowsValueIs}(\langle 0\rangle, \mathit{Collected}, 4, \mathit{True}).$$
$$\neg\mathit{KnowsValue}(\langle 2\rangle, \mathit{CollectionPoint}, 2).$$
$$\mathit{KnowsValue}(\langle 3\rangle, \mathit{CollectionPoint}, 2).$$
$$\mathit{KnowsValue}(\langle 3\rangle, \mathit{CollectionPoint}, 3).$$
$$\mathit{KnowsHappens}(\langle 5\rangle, \mathit{Purchase}, 1).$$
$$\mathit{KnowsIfHappens}(\langle 5\rangle, \mathit{GiveFreeGift}, 3).$$

Another brief example of sensing evidence about the past: *A disease triggers the production of antibodies that cure the disease and remain in the bloodstream afterwards. A person knows she did not have the antibodies at time* 0 *but wonders if she had just contracted the disease at that time. At time* 1 *she can ascertain this by testing (sensing) for the antibodies.* Assuming $\mathcal{I} = \mathbb{N}$, fluents are truth-valued, and domain closure and uniqueness-of-names axioms DC_A and UNA_A (analagous to (S1)-(S8)) our representation is:

$$KnowsValueIsNot(\langle 0\rangle, f, 0, v) \equiv [f = Antibodies \wedge v = True]. \quad \text{(A1)}$$

$$CausesValue(a, f, v, t) \equiv [(a = MakeAntibodies \wedge f = Antibodies \wedge v = True) \vee (a = MakeAntibodies \wedge f = Disease \wedge v = False) \vee \exists f', w, w', i (a = Sense(f') \wedge f = K(w') \wedge v = False \wedge t = \langle w, i\rangle \wedge ValueOf(f', \langle w, i\rangle) \neq ValueOf(f', \langle w', i\rangle))]. \quad \text{(A2)}$$

$$Triggered(a, t) \equiv [a = MakeAntibodies \wedge ValueOf(Disease, t) = True]. \quad \text{(A3)}$$

$$Perform(a, i) \equiv [a = Sense(Antibodies) \wedge i = 1]. \quad \text{(A4)}$$

$$\neg PerformIfValueKnownIs(a, f, v, i). \quad \text{(A5)}$$

Proposition 2. The theory (FEC1)$\wedge$...$\wedge$(FEC5)$\wedge$(EFEC1)$\wedge$ ... $\wedge$ (EFEC26) $\wedge$ DC_A $\wedge$ UNA_A $\wedge$ (A1) $\wedge$ ... $\wedge$ (A5) entails the following sentences:

$$\neg KnowsValue(\langle 0\rangle, Disease, 0).$$
$$KnowsValue(\langle 2\rangle, Disease, 0).$$
$$\neg KnowsIfHappens(\langle 0\rangle, MakeAntibodies, 0).$$
$$KnowsIfHappens(\langle 2\rangle, MakeAntibodies, 0).$$

Fluent Formulae for Complex Conditions

For the shopping domain, it is sufficient for the 2nd argument in *KnowsValueIs* and *PerformIfValueKnownIs* to be a single fluent. For more complex conditions, we introduce a sort $\mathcal{G}$ of *fluent formulae*. Space restrictions forbid a full treatment here, but the following predicate definitions briefly illustrate:

$$HoldsFormula(f \tilde{=} v, t) \equiv ValueOf(f, t) = v. \quad \text{(EFEC25)}$$

$$HoldsFormula(\tilde{\neg} g, t) \equiv \neg HoldsFormula(g, t). \quad \text{(EFEC26)}$$

$$HoldsFormula(g \tilde{\wedge} g', t) \equiv (HoldsFormula(g, t) \wedge HoldsFormula(g', t)). \quad \text{(EFEC27)}$$

$$KnowsHoldsFormula(\langle w, i\rangle, g, i') \equiv \forall w' [ValueOf(K(w'), \langle w, i\rangle) = True \rightarrow HoldsFormula(g, \langle w', i'\rangle)]. \quad \text{(EFEC28)}$$

$$PerformIfKnowsHoldsFormula(a, g, i) \rightarrow [KnowsHoldsFormula(\langle w, i\rangle, g, i) \equiv Happens(a, \langle w, i\rangle)]. \quad \text{(EFEC29)}$$

4 Summary, Related and Future Work

The contributions of this paper are (i) to generalize the EC of (Miller and Shanahan 2002) to multi-valued (non-binary) fluents, and (ii) to build upon this generalization to provide an EC framework for combined narrative, epistemic and causal reasoning under a possible-worlds approach. EFEC is able to deal with triggered, concurrent, non-deterministic and conflicting action occurrences in a uniform manner under both discrete and continuous models of time. It facilitates reasoning about knowledge of both action occurrences and fluent values at past, present and future times, as well as epistemically feasible planning, where conditional actions' conditions are guaranteed to be known by the time of potential execution. To the best of our knowledge, no other existing epistemic action formalism is able to deal with this combination of features. In particular, triggered events (and knowledge about them) have not previously been incorporated in epistemic reasoning frameworks. This is in spite of their recognised importance in modelling many domains, e.g. involving complex ramifications (Mueller 2006), or reasoning about biological, physical or mechanical systems (Tran and Baral 2004; Miller and Shanahan 1996), and many modes of reasoning, e.g. evidence gathering, diagnosis, scientific investigation.

Our work is related to, and inspired by, the work of Scherl and Levesque [2003; 1993], who used possible situations to specify how the mental state of an agent should change with ordinary and sense actions, providing a solution to the frame problem for knowledge. It evolved from Moore's [1985] Kripke-like formulation of epistemic notions of modal logic in action theories by reifying possible worlds as situations. Since then several other studies have extended this model with new features: (Thielscher 2000) adapted the model in the context of the Fluent Calculus, (Scherl 2003) covered concurrent actions, (Kelly and Pearce 2008) introduced epistemic modalities for groups of agents, and (Shapiro et al. 2011) extended the model to account for belief revision.

To our knowledge little work has previously been done in extending possible-worlds epistemic action theories to deal with non-deterministic actions (resulting in knowledge loss). (An exception is $\mathcal{A}_k$ (Lobo, Mendez, and Taylor 2001) that also accounts for conditional sensing, but not functional fluents, concurrent actions, narrative reasoning or triggered events.) This may in part be explained by the technical difficulty of ensuring that in each model there are sufficient possible worlds to properly model the lack of knowledge that ensues after a non-deterministic event. In our framework this has been made possible partly because non-deterministic effects are represented as conjuncts (of *CausesValue*) rather than disjuncts, so that in each model each conjunct can be associated with an accessible world. We see no reason why this solution should not be translated into Situation Calculus-related approaches.

Non-determinism has been studied in non-possible-worlds approaches, e.g. (Baldoni et al. 2004; Eiter et al. 2004). Epistemic action frameworks that use alternative models to represent knowledge also include (Morgenstern 1987; Demolombe and Pozos-Parra 2000; Son and Baral 2001; Petrick and Levesque 2002; Liu and Levesque 2005; Vassos and Levesque 2007; Liu and Lakemeyer 2009; Patkos and Plexousakis 2009). This last work is also EC-based. Although it is limited to discrete time and binary fluents, it can model ramifications, a feature we have not yet investigated.

Other than considering ramifications, our future plans include more formal or general methods of showing the correctness and limitations of our approach (other than case studies) and its correspondence with other frameworks, further work on modeling both hypothetical and explicit knowledge about the future (e.g. drawing on (Davis and Morgenstern 2005)), consideration of belief (and belief revision) rather than knowledge, nested knowledge/belief structures, multi-agent domains, and implementation, e.g. using ASP (Baral 2003). Developments in our EFEC-related research will be documented at `http://www.ucl.ac.uk/infostudies/efec`.

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