An Argumentation Framework for Merging Conflicting Knowledge Bases: The Prioritized Case

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Abstract

An important problem in the management of knowledge-based systems is the handling of inconsistency. Inconsistency may appear because the knowledge may come from different sources of information. To solve this problem, two kinds of approaches have been proposed. The first category *merges* the different bases into a unique base, and the second category of approaches, such as argumentation, accepts inconsistency and copes with it.

Recently, a "powerful" approach [Benferhat *et al.*, 2002; 1999; Kaci, 2002] has been proposed to merge *prioritized* propositional bases encoded in possibilistic logic. This approach consists of combining prioritized knowledge bases into a new prioritized knowledge base, and then to infer from this.

In this paper, we present a *particular* argumentation framework for handling inconsistency arising from the presence of multiple sources of information. Then, we will show that this framework retrieves the results of the merging operator defined in [Benferhat *et al.*, 2002; 1999; Kaci, 2002]. Moreover, we will show that an argumentation-based approach palliates the limits, due to the *drowning* problem, of the merging operator.

Keywords: Argumentation, Information merging, Possibilistic logic.

1 Introduction

In many areas such as cooperative information systems, multi-databases, multi-agents reasoning systems, GroupWare, distributed expert systems, information comes from multiple sources. The multiplicity of sources providing information makes that information is often contradictory and the use of priorities is crucial to solve conflicts.

We distinguish two approaches to deal with contradictory information coming from multiple sources:

- The first approach consists of merging these items of information and constructing a consistent set of information which represents the result of merging [Cholvy, 1998; Konieczny and Pérez, 1998; Lin, 1996; Lin and Mendelzon, 1998; Rescher and Manor, 1970; Revesz, 1993]. In other words, starting from different bases $B_1, \cdots, B_n$ which are conflicting, these works return a *unique consistent base*. Several approaches have been proposed for merging multiple sources of information where priorities are either implicitly [Konieczny and Pérez, 1998; Lin, 1996; Lin and Mendelzon, 1998; Rescher and Manor, 1970; Revesz, 1993] or explicitly expressed [Benferhat *et al.*, 2002; 1999; Kaci, 2002]. Possibilistic logic [Dubois *et al.*, 1994; Lang, 2000] is a suitable framework for modeling explicit priorities. It is an extension of classical logic which allows to model prioritized information encoded by means of weighted propositional formulas. Possibilistic logic has a syntactic inference which is sound and complete w.r.t. semantics based on the notion of possibility distributions [Dubois *et al.*, 1994]. Merging prioritized information in this framework turns out to build from sets of prioritized information a new set of prioritized information, from which inferences are drawn.

- The second approach consists of solving the conflicts without merging the bases. Argumentation is one of the most promising of these approaches. It is based on the construction of arguments and counter-arguments (defeaters) and the selection of the most acceptable of these arguments.

The present paper completes the results presented in [Amgoud and Parsons, 2002] where the *relationship* between information merging, when priorities are implicitly expressed, and argumentation theory has been established.

In this paper, we consider the case of priorities expressed *explicitly* in a possibilistic logic framework. We will show that the results of the merging operator defined in [Benferhat *et al.*, 2002; 1999; Kaci, 2002] are retrieved in a *particular* argumentation framework. In that framework, the arguments are built from the different bases, and each argument has an *intrinsic force* based on the certainty level of the information used in that argument. Moreover, we will show that an argumentation-based approach palliates the limits, due to the *drowning* problem, of the merging operator. All the proofs of the results given in this paper can be found in [Amgoud and Kaci, 2005].

The paper is organized as follows: section 2 recalls briefly the basics of possibilistic logic. Section 3 introduces a merging operator based on possibilitic logic. In section 4 a general preference-based argumentation framework is presented. Section 5 connects argumentation theory with the merging operator defined in section 3. Section 6 is devoted to some concluding remarks and perspectives.

2 Brief refresher on possibilistic logic

Let us consider a propositional language $\mathcal{L}$ over a finite alphabet $\mathcal{P}$ of atoms. Ω denotes the set of all the interpretations. Logical equivalence is denoted by $\equiv$ and classical conjunction and disjunction are respectively denoted by $\wedge$, $\vee$. $\vdash$ denotes classical inference. The notation $\omega \models \phi$ means that the interpretation ω is a model of (or satisfies) the formula ϕ.

At the semantic level, possibilistic logic is based on the notion of a possibility distribution [Zadeh, 1978], denoted by π, which is a mapping from Ω to [0,1] representing the available information. $\pi(\omega)$ represents the degree of compatibility of the interpretation ω with the available beliefs about the real world if we are representing uncertain pieces of knowledge (or the degree of satisfaction of reaching a state ω if we are modeling preferences). By convention, $\pi(\omega) = 1$ means that it is totally possible for ω to be the real world (or that ω is fully satisfactory), $1 > \pi(\omega) > 0$ means that ω is only somewhat possible (or satisfactory), while $\pi(\omega) = 0$ means that ω is certainly not the real world (or not satisfactory at all). Associated with a possibility distribution π is the necessity degree of any formula ϕ: $N(\phi) = 1 - \Pi(\neg\phi)$ which evaluates to what extent ϕ is entailed by the available beliefs, and defined from the consistency degree of a formula ϕ w.r.t. the available information, $\Pi(\phi) = max\{\pi(\omega) : \omega \models \Omega \text{ and } \omega \models \phi\}$.
Note that the mapping N reverses the scale on which π is ranging, and that $N(\phi) = 1$ means that ϕ is a totally certain piece of knowledge or a compulsory goal, while $N(\phi) = 0$ expresses the complete lack of knowledge or of priority about ϕ, but does not mean that ϕ is or should be false. Moreover, the duality equation $N(\phi) = 1 - \Pi(\neg\phi)$ extends the existing one in classical logic, where a formula is entailed from a set of classical formulas if and only if its negation is consistent with this set.

At the syntactic level, prioritized items of information are represented by means of *a possibilistic knowledge base* (or *a possibilistic base* for short) which is a set of weighted formulas of the form $B = \{(\phi_i, a_i) : i = 1, \cdots, n\}$, where ϕ_i is a propositional formula and a_i belongs to a totally ordered scale such as [0,1]. The pair (ϕ_i, a_i) means that the certainty degree of ϕ_i is at least equal to a_i ($N(\phi_i) \geq a_i$). We denote by B^* the propositional base associated with B, namely the base obtained from B by forgetting the weights of formulas. A possibilistic base B is consistent if and only if its associated propositional base B^* is consistent.
Given a possibilistic base B, we can generate a unique possibility distribution, denoted by π_B, such that all the interpretations satisfying all the formulas in B will have the highest possibility degree, namely 1, and the other interpretations will be ranked w.r.t. the highest formula that they falsify, namely we get [Dubois *et al.*, 1994]:

Definition 1 $\forall \omega \in \Omega$,

$$\pi_B(\omega) = \begin{cases} 1 & \textit{if } \forall(\phi_i, a_i) \in B, \omega \models \phi_i \\ 1 - max\{a_i : (\phi_i, a_i) \in B \textit{ and } \omega \not\models \phi_i\} & \textit{otherwise.} \end{cases}$$

Example 1 *Let $B = \{(\neg p \vee \neg q, .7); (p, .6)\}$ be a knowledge base. Its associated possibility distribution is: $\pi_B(p\neg q) = 1$; $\pi_B(\neg p \neg q) = \pi_B(\neg pq) = .4$ and $\pi_B(pq) = .3$.*
The interpretation $p\neg q$ is the most preferred since it satisfies all the formulas in B. The interpretations $\neg p\neg q$ and $\neg pq$ are more preferred than pq since the highest formula falsified by $\neg p \neg q$ and $\neg pq$ (i.e., $(p, .6)$) is less certain (or less prioritized) than the highest formula falsified by pq (i.e., $(\neg p \vee \neg q, .7)$).

In the following, we give some definitions useful for the rest of the paper:

Definition 2 *Let B_1 and B_2 be two possibilistic bases. B_1 and B_2 are said to be* equivalent, *denoted by $B_1 \equiv_s B_2$, iff $\pi_{B_1} = \pi_{B_2}$.*

Definition 3 (a-cut and strict a-cut) *Let B be a possibilistic knowledge base, and $\mathbf{a} \in [0, 1]$. We call the $\mathbf{a}$-cut (resp. strict $\mathbf{a}$-cut) of B, denoted by $B_{\geq \mathbf{a}}$ (resp. $B_{>\mathbf{a}}$), the set of propositional formulas in B having a certainty degree at least equal to $\mathbf{a}$ (resp. strictly greater than $\mathbf{a}$).*

Definition 4 (Inconsistency degree) *The* inconsistency degree *of a possibilistic base B is:*

$$Inc(B) = max\{a_i : B_{\geq a_i} \textit{ is inconsistent}\},$$

with $Inc(B) = 0$ when B is consistent.

Definition 5 (Subsumption) *Let (ϕ, a) be a formula in B. (ϕ, a) is said to be* subsumed *in B if:*

$$(B - \{(\phi, a)\})_{\geq a} \vdash \phi.$$

And (ϕ, a) is said to be strictly subsumed in B if $B_{>a} \vdash \phi$.

Subsumed formulas are in some sense redundant formulas as it is shown by the following lemma [Benferhat *et al.*, 1999]:

Lemma 1 *Let (ϕ, a) be a subsumed formula in B. Then B and $B' = B - \{(\phi, a)\}$ are equivalent.*

Lastly, weights are propagated out in the inference process in the following way:

Definition 6 (Plausible inference) *Let B be a possibilistic base. The formula ϕ is a* plausible consequence *of B iff*

$$B_{>Inc(B)} \vdash \phi.$$

Definition 7 (Possibilistic inference) *Let B be a possibilistic base. The formula (ϕ, a) is a* possibilistic consequence *of B, denoted $B \vdash_\pi (\phi, a)$, iff*

- $B_{>Inc(B)} \vdash \phi$,
- $a > Inc(B)$ *and* $\forall b > a$, $B_{>b} \not\vdash \phi$.

3 Merging prioritized information in possibilistic logic framework

Merging prioritized information in possibilistic logic is a two step process:

1. From a set of possibilistic bases[1], computing a new possibilistic base, called the *aggregated base*, which is generally inconsistent [Benferhat *et al.*, 1999].
2. Inferring conclusions from the new base.

A possibilistic merging operator, denoted by $\oplus$, is a function from $[0,1]^n$ to $[0,1]$. $\oplus$ is used to aggregate the certainty degrees associated with pieces of information provided by different sources. Formally, let $\mathcal{B} = \{B_1, \cdots, B_n\}$ be a set of n (possibly inconsistent) possibilistic bases. The result of merging the bases of $\mathcal{B}$ using $\oplus$, denoted by $\mathcal{B}_\oplus$, is defined as follows [Benferhat *et al.*, 2002]:

Definition 8 (Aggregated base) *Let $\mathcal{B}= \{B_1, \cdots, B_n\}$ be a set of possibilistic bases and $\oplus$ a merging operator. The result of merging $\mathcal{B}$ with $\oplus$ is defined by:*

$$\mathcal{B}_\oplus = \{(D_j, \oplus(x_1, \cdots, x_n)) : j = 1, \cdots, n\},$$

where D_j are disjunctions of size j between formulas taken from different B_i's ($i = 1, \cdots, n$) and x_i is either equal to a_i or to 0 depending respectively on whether ϕ_i belongs to D_j or not.

Two properties for $\oplus$ are assumed in this definition [Benferhat and Kaci, 2003; Benferhat *et al.*, 1999]:

1. $\oplus(0, \cdots, 0) = 0$,
2. If $a_i \geq b_i$ for all $i = 1, \cdots, n$ then $\oplus(a_1, \cdots, a_n) \geq \oplus(b_1, \cdots, b_n)$.

The first property says that if a formula doesn't explicitly appear in any base, then it should not appear explicitly in the result of merging. The second property is simply the unanimity property (called also monotonicity property) which means that if all the sources say that a formula ϕ is more plausible than (or preferred to) another formula ψ, then the result of merging should confirm this preference.

Example 2 *Let $B_1 = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\xi, .1)\}$ and $B_2 = \{(\neg\psi, .7), (\phi, .6)\}$. Let $\oplus$ be the probabilistic sum defined by $\oplus(a, b) = a + b - ab$. Following Definition 8, we get:*
$\mathcal{B}_\oplus = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\xi, .1)\} \cup \{(\neg\psi, .7), (\phi, .6)\} \cup \{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94),$
$(\xi \vee \neg\psi, .73), (\xi \vee \phi, .64)\}$ *which is equivalent to*
$\{(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94), (\neg\phi, .8), (\xi \vee \neg\psi, .73),$
$(\neg\psi, .7), (\xi \vee \phi, .64), (\phi, .6), (\xi, .1)\}$.

Lemma 2 gives a rewriting of $\mathcal{B}_\oplus$ given in Definition 8 which will be useful in the rest of the paper, but first let us give the following definition:

Definition 9 (Existential consequence) *Let B be a possibilistic base. The formula (ϕ, a) is an* existential consequence *of B, denoted by $B \Vdash (\phi, a)$, iff:*

1. *$\exists B' \subseteq B$ s.t. $B' \vdash_\pi (\phi, a)$,*
2. *B' is consistent,*
3. *$a = min\{a_i : (\phi_i, a_i) \in B'\}$,*
4. *B' is a minimal for set inclusion,*
5. *$\nexists B'' \subseteq B$ satisfying the above conditions with $B'' \vdash_\pi (\phi, b)$ and $b > a$.*

This definition focuses on the subbases containing the most prioritized formulas.

Example 3 *Let $B = \{(\phi \vee \psi, .9), (\neg\phi, .7), (\xi \vee \psi, .6), (\neg\xi, .5)\}$. Then $B \Vdash (\phi \vee \psi, .9)$, $B \Vdash (\neg\phi, .7)$ and $B \Vdash (\psi, .7)$ however $B \Vdash (\neg\psi, 0)$.*

Lemma 2 *Let $\mathcal{B}_\oplus$ be the result of merging $\mathcal{B} = \{B_1, \cdots, B_n\}$ with $\oplus$. Then, $\mathcal{B}_\oplus$ is equivalent to*

$$\{(\phi, \oplus(a_1, \cdots, a_n)) : \phi \in \mathcal{L} \text{ and } B_i \Vdash (\phi, a_i)\}.$$

Now that the base $\mathcal{B}_\oplus$ is defined, we are ready to define the result of merging. This corresponds to the possibilistic consequences of $\mathcal{B}_\oplus$. Formally:

Definition 10 (Useful result of merging) *Let $\mathcal{B}_1 = \{B_1, \cdots, B_n\}$ be a set of n possibilistic bases, $\oplus$ be a merging operator and $\mathcal{B}_\oplus$ be the result of merging $\mathcal{B}$ with $\oplus$. The useful result of merging is:*

$$\mathcal{T} = \{(\phi_i, a_i) \mid \mathcal{B}_\oplus \vdash_\pi (\phi_i, a_i)\}.$$

4 Basic argumentation framework

Argumentation is a reasoning model based on the construction and the comparison of arguments. Argumentation frameworks have been developed for decision making under uncertainty [Amgoud and Prade, 2004], and for handling inconsistency in knowledge bases where each conclusion is justified by arguments [Amgoud and Cayrol, 2002a; Prakken and Sartor, 1997]. Arguments represent the reasons to believe in a fact. In what follows, we present the general framework proposed in [Amgoud and Cayrol, 2002b] which is an extension of the famous framework presented by Dung in [Dung, 1995].

Definition 11 (Argumentation framework) *An* argumentation framework *(AF) is a triplet $\langle\mathcal{A}, \mathcal{R}, \succeq\rangle$, where $\mathcal{A}$ is a set arguments, $\mathcal{R}$ is a binary relation representing defeat relationship between arguments and $\succeq$ is a (partial or complete) pre-ordering on $\mathcal{A} \times \mathcal{A}$. The strict ordering associated with $\succeq$ is denoted $\succ$.*

Since arguments are conflicting, it is important to define the acceptable ones (i.e the "good" ones). Different semantics have been introduced in [Dung, 1995]. In what follows, we will focus only on one of them, the so-called *grounded extension*.

The preference order between arguments makes it possible to distinguish different types of relations between arguments:

Definition 12 *Let A, B be two arguments of $\mathcal{A}$.*

- *B* attacks *A iff $B \mathcal{R} A$ and it is not the case that $A \succ B$.*
- *If $B \mathcal{R} A$ then A* defends itself *against B iff $A \succ B$.*
- *A set of arguments $\mathcal{S}$* defends *A if there is some argument in $\mathcal{S}$ which attacks every argument B where B attacks A.*

[1]These bases may be individually inconsistent.

Henceforth, $C_{\mathcal{R},\succeq}$ will gather all non-defeated arguments and arguments defending themselves against all their defeaters. In [Amgoud and Cayrol, 2002b], it has been shown that the set $\underline{\mathcal{S}}$ of acceptable arguments of the argumentation framework $\langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ is the least fixpoint of a function $\mathcal{F}$:

$$\begin{aligned} \mathcal{S} &\subseteq \mathcal{A} \\ \mathcal{F}(\mathcal{S}) &= \{A \in \mathcal{A} | A \text{ is defended by } \mathcal{S}\}. \end{aligned}$$

Definition 13 *The set of* acceptable *arguments for an argumentation framework $\langle \mathcal{A}, \mathcal{R}, \succeq \rangle$ is:*

$$\begin{aligned} \underline{\mathcal{S}} &= \mathcal{B}igcup \mathcal{F}_{i \geq 0}(\emptyset) \\ &= C_{\mathcal{R},\succeq} \cup [\mathcal{B}igcup \mathcal{F}_{i\geq 1}(C_{\mathcal{R},\succeq})] \,. \end{aligned}$$

An argument is acceptable *if it is a member of the acceptable set.*

5 Relating merging in possibilistic logic with argumentation

In section 4, we have introduced a general argumentation framework. In that framework, the structure and the origin of arguments are not defined. Similarly, the defeasibility and the preference relations between arguments are not given too. In what follows, we will give an instantiation of the above framework for handling inconsistency in knowledge bases, especially when the inconsistency occurs because of the presence of different and conflicting sources of information (let's say, $B_1, \ldots, B_n$). We will then show that the obtained system retrieves the results of the merging operator introduced in section 3.

Let's first recall some concepts. Let B_1, ..., B_n be different possibilistic bases. $Disj$ will denote the set of all disjunctions of different size that can be formed from formulas of the n bases. $Conj$ will denote the set of formulas of B_1, ..., B_n with possibly new weights. Weights of formulas in $Disj$ and $Conj$ are aggregated using an operator $\otimes$. For instance, if the formula (ϕ, a) is in B_1 and (ψ, b) is in B_2, then the formula ($\phi \vee \psi$, $\otimes$(a, b)) will be in $Disj$ and the formulas $(\phi, \otimes(a, 0))$ and $(\psi, \otimes(0, b))$ will be in $Conj$, with $\otimes$(x, y) is max(x, y) or min(x, y) etc. In what follows, $\mathcal{B} = Conj \cup Disj$. In fact, it can be shown that if the aggregation operator $\otimes$ is exactly the operator $\oplus$, then the two bases $\mathcal{B}$ and $\mathcal{B}_{\oplus}$ are equivalent.

Proposition 1 *Let B_1, ..., B_n be different possibilistic bases. If $\otimes = \oplus$, then the bases $\mathcal{B}$ and $\mathcal{B}_{\oplus}$ are equivalent.*

Let's start now by defining the notion of argument. An argument has a deductive form and takes the form of an explanation. Each argument is constructed from formulas of $B_1, \cdots, B_n$ and disjunctions between formulas of these bases.

Definition 14 (Argument) *An* argument *is a pair $<H, h>$, where h is a formula of the language $\mathcal{L}$ and H a subset of B satisfying:*

1. *$H \subseteq \mathcal{B}^*$,*
2. *$H \vdash h$,*
3. *H is consistent and minimal (no strict subset of H satisfies 1 and 2).*

H is called the support *and h the* conclusion *of the argument. $\mathcal{A}(\mathcal{B})$ will denote the set of all arguments that can be built from $\mathcal{B}$.*

Note that it is not necessary to construct the bases $Disj$ and $Conj$ in order to define the arguments. Fragments of these bases are constructed only when needed i.e., when building arguments.

The most appropriate defeat relation which will capture all the different kinds of conflicts which may exist between arguments is the following relation "undercut".

Definition 15 (Undercut relation) *Let $<H, h>$ and $<H', h'>$ be two arguments of $\mathcal{A}(\mathcal{B})$. $<H, h>$* undercuts *$<H', h'>$ iff for some $k \in H'$, $h \equiv \neg k$. An argument is undercut if there exists at least one argument against one element of its support.*

In [Amgoud and Cayrol, 2002a], it has been argued that arguments may have forces of various strengths. These forces allow an agent to compare different arguments in order to select the 'best' ones.
When explicit priorities are given between the beliefs, such as certainty degrees, the arguments using more certain beliefs are found stronger than arguments using less certain beliefs. The force of an argument corresponds to the *certainty degree* of the less entrenched belief involved in the argument.

Definition 16 (Force of an argument) *Let $A = <H, h>$ be an argument. The* force *of A, denoted by $force(A)$, is*

$$force(A) = min\{a_i : \phi_i \in H \text{ and } (\phi_i, a_i) \in \mathcal{B}\}.$$

The following proposition shows that an argument and its force can be constructed from $\mathcal{B}$ without computing explicitly the base $Disj$.

Proposition 2 *Let $B_1, \cdots, B_n$ be n possibilistic bases. Let $A = <H, \phi>$ be an argument in $\mathcal{A}(\mathcal{B})$. It holds that:*

- *$\forall\ \phi_j \in H$, $B_i \Vdash (\phi_j, a_{ji})$, i=1, $\cdots$, n.*
- *force(A) = min$\{\otimes(a_{j1}, \cdots, a_{jn})$ with and $a_j = \otimes(a_{j1}, \cdots, a_{jn})\}$.*

Example 4 *Let's compute an argument for $\phi \vee \psi$ from $\mathcal{B}_{\oplus}$. We get $A_1 =< \{\phi \vee \psi\}, \phi \vee \psi >$ and $A_2 =< \{\phi\}, \phi \vee \psi >$. A_1 is stronger than A_2 since $force(A_1) = .96$ whereas $force(A_2) = .6$.*
Now $B_1 \Vdash (\phi \vee \psi, .9)$ and $B_2 \Vdash (\phi \vee \psi, .6)$. Then, $force(A_1) = min\{\oplus(.9, .6)\} = .96$.

The forces of arguments make it possible to compare pairs of arguments as follows:

Definition 17 (Preference relation) *Let A and A' be two arguments in $\mathcal{A}(\mathcal{B})$. A is* preferred *to A', denoted by $A \succ A'$, iff $force(A) > force(A')$.*

Example 5 *Let us consider again the possibilistic base given in Example 3: $B = \{(\phi \vee \psi, .9), (\neg\phi, .7), (\xi \vee \psi, .6), (\neg\xi, .5)\}$. There are two arguments in favor of ψ:*

- *$A_1 = <\{\phi \vee \psi, \neg\phi\}, \psi>$,*
- *$A_2 = <\{\xi \vee \psi, \neg\xi\}, \psi>$.*

However, it is clear that A_1 is preferred to A_2 since $force(A_1) = .7$ whereas $force(A_2) = .5$.

Definition 18 (Acceptable arguments) *Let $<\mathcal{A}(\mathcal{B}), Undercut, \succ>$ be an argumentation framework. Its set of* acceptable *arguments is:*

$$\begin{aligned} \underline{\mathcal{S}} &= \mathcal{B}igcup\mathcal{F}_{i\geq 0}(\emptyset) \\ &= C_{Undercut,\succ} \cup [\mathcal{B}igcup\mathcal{F}_{i\geq 1}(C_{Undercut,\succ})] \end{aligned}$$

An important result states that the obtained set of acceptable arguments is not conflicting. Moreover, the set of formulas that constitute that set of acceptable arguments is consistent.

Definition 19 *Let $T \subseteq \mathcal{A}(\mathcal{B})$. $Supp(T) = \cup H_i$ such that $<H_i, h_i> \in T$.*

Proposition 3 *Let $\langle \mathcal{A}(\mathcal{B}), Undercut, \succ\rangle$ be an argumentation framework.*

1. *$\nexists$ A, B $\in \underline{\mathcal{S}}$ such that A undercuts B.*
2. *Supp($\underline{\mathcal{S}}$) is consistent.*

We can show easily that any plausible consequence of a given possibilistic base B_i is supported by an acceptable argument, if we consider only the arguments $\mathcal{A}(B_i)$ built from that base B_i.

Proposition 4 *Let B_i be a possibilistic base, and let $\langle \mathcal{A}(B_i)$, $Undercut, \succ\rangle$ be an argumentation framework and $\underline{\mathcal{S}}$ its set of acceptable arguments.*
If ϕ is a plausible consequence of B_i, then $\exists\ A = <H, \phi> \in \underline{\mathcal{S}}$.

Another interesting result states that any possibilistic consequence (ϕ, a) of a given possibilistic base B_i is supported by an acceptable argument A whose force is equal to a. Moreover, A is the strongest argument w.r.t $\succ$ in favor of ϕ. This means that the degree a of a possibilistic consequence ϕ corresponds to the force of the best argument in favor of ϕ.

Proposition 5 *Let B_i be a possibilistic base, and let $\langle \mathcal{A}(B_i)$, $Undercut, \succ\rangle$ be an argumentation framework and $\underline{\mathcal{S}}$ its set of acceptable arguments.*
If (ϕ, a) is a possibilistic consequence of B_i, then $\exists\ A = <H, \phi> \in \underline{\mathcal{S}}$ with $force(A) = a$, and $\forall\ A' = <H', \phi> \in \underline{\mathcal{S}}$, $A \succ A'$.

An important concept in possibilistic logic is that of *inconsistency degree* of a possibilistic base B_i. In what follows, we will show that that inconsistency degree can be computed from the forces of the conflicting arguments as follows:

Proposition 6 *Let B be a possibilistic base, and let $\langle \mathcal{A}(B)$, $Undercut, \succ\rangle$ be an argumentation framework.*

$$Inc(B) = max\{min(force(A_i), force(A_j)) \mid A_i\ undercuts\ A_j\}$$

Example 6 *Let's consider the base $\mathcal{B}_\oplus$ constructed in Example 2: $\mathcal{B}_\oplus$ = {($\phi \vee \psi$,.96), ($\neg\phi \vee \neg\psi$,.94), ($\neg\phi$,.8), ($\xi \vee \neg\psi$,.73), ($\neg\psi$,.7), ($\xi \vee \phi$,.64), (ϕ,.6), (ξ,.1)}.*
Table 1 summarizes the different arguments which can be constructed from $\mathcal{B}_\oplus$ and their force. As we mentioned before, note that we only focus on the best arguments (i.e., having the highest force) in favor of formulas. For example, there is an argument $A =< \{\phi\}, \phi \vee \psi >$, with a force equal to .6, in favor of $\phi \vee \psi$ however it is not considered since there is another argument A_1 in favor of $\phi \vee \psi$ with a higher force.

Argument	Force
$A_1 =< \{\phi \vee \psi\}, \phi \vee \psi >$	.96
$A_2 =< \{\neg\phi \vee \neg\psi\}, \neg\phi \vee \neg\psi >$	.94
$A_3 =< \{\neg\phi\}, \neg\phi >$	.8
$A_4 =< \{\xi \vee \neg\psi, \neg\phi, \phi \vee \psi\}, \xi >$	.73
$A_5 =< \{\neg\psi\}, \neg\psi >$	.7
$A_6 =< \{\phi \vee \psi, \neg\psi\}, \phi >$	.7
$A_7 =< \{\neg\phi, \phi \vee \psi\}, \psi >$	.8

Table 1:

We have $Undercut = \{(A_6, A_3), (A_6, A_4), (A_7, A_5), (A_7, A_6), (A_6, A_7)\}$.
Then, max{min(.7,.8), min(.7,.73), min(.8,.7), min(.8,.7), min(.7,.8)} = .7. It can be checked that the inconsistency degree of $\mathcal{B}_\oplus$ is .7.

Indeed we have the following result:

Proposition 7 *Let B be a possibilistic base.*

1. *A formula ϕ is a plausible consequence of B iff $\exists\ A = <H, \phi>$ in $\mathcal{A}(B)$ s.t. $force(A) > Inc(B)$.*
2. *A formula (ϕ, a) is a possibilistic consequence of B iff $\exists\ A = <H, \phi>$ in $\mathcal{A}(B)$ s.t. $force(A) > Inc(B)$ and $force(A) = a$.*

Example 7 *Let's consider the different arguments of Example 6. Only the arguments having a weight strictly greater than .7 are considered. Namely A_1, A_2, A_3, A_4 and A_7. Thus, the plausible consequences of $\mathcal{B}_\oplus$ are $\phi \vee \psi, \neg\phi \vee \neg\psi, \neg\phi, \xi$ and ψ. The possibilistic consequences of $\mathcal{B}_\oplus$ are $(\phi \vee \psi, .96), (\neg\phi \vee \neg\psi, .94)$, $(\neg\phi, .8), (\xi, .73)$ and $(\psi, .8)$.*

From the previous propositions, it can be shown that the result of merging is captured in argumentation framework. Formally:

Theorem 1 *Let $B_1, \cdots, B_n$ different possibilistic bases, and $\langle \mathcal{A}, Undercut, \succeq\rangle$ be an argumentation framework. If $\oplus = \otimes$ then the following result holds:*

$$\mathcal{T} \subseteq Supp(\underline{\mathcal{S}}),$$

where $\mathcal{T}$ is given in Definition 10.

The above result shows that an argumentation framework is "stronger" than the merging operator defined in section 3 in the sense that it may return more results. The reason is that possibilistic logic suffers from the so-called *drowning* problem. A drowning problem means that some information which are not responsible of conflicts may be ignored [Benferhat *et al.*, 1993]. More precisely, formulas at the level and below the inconsistency degree are ignored.

Example 8 *Let us consider again the bases B_1 and B_2 given in Example 2. Let $\oplus$ be the max operator. Then, $\mathcal{B}_\oplus = B_1 \cup B_2 = \{(\phi \vee \psi, .9), (\neg\phi, .8), (\neg\psi, .7), (\phi, .6), (\xi, .1)\}$.*
Using the inference in possibilistic logic, plausible consequences are $\phi \vee \psi, \neg\phi$ and ψ while the argumentation-based inference gives $\{\phi \vee \psi, \neg\phi, \psi, \xi\}$.

6 Conclusion

We presented in this paper an argumentation-based framework for resolving conflicts between knowledge bases in a prioritized case where priorities are represented in possibilistic logic framework. The proposed approach is different from the classical way used in the literature to deal with conflicting multiple sources information.

The classical existing approaches consist of first merging individual bases into a new base from which conclusions are drawn. The new base is composed of the most prioritized consistent formulas. The drawback of this approach is that it may ignore formulas which are not responsible for the conflicts.

The argumentation-based approach proposed here builds arguments from the separate bases, evaluates them and lastly computes a set of acceptable arguments from which conclusions are drawn.

The main result of the work presented in this paper is that the argumentation framework captures the result of the merging operator defined in [Benferhat *et al.*, 2002; 1999; Kaci, 2002] without merging the different bases. This is of great importance since merging the bases is computationally very costly. Moreover, it is not always interesting to merge the bases as it is the case in a multi-agent system. In such a system, each agent has its own base which may conflict with the bases of the other agents.

We have shown also that the argumentation-based framework solves the drowning problem. Consequently, it returns more formulas than the approach which merges the bases.

An extension of this work would be to study the behavior of the argumentation-based approach proposed in this paper from a postulate point of view inspired from the description of possibilistic merging operators from postulate point of view given in [Benferhat and Kaci, 2003]. We are also planning to investigate how argumentation framework can capture the result of merging when multiple-operators are used as in [Qi *et al.*, 2004]. In that work, two merging operators are used for consistent and conflicting formulas respectively. Another extension consists of comparing the argumentation-based approach and the merging-based approach from a complexity in space and time point of view.

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